A Note on "Surface Roughness Prediction Model in Machining of Carbon Steel by PVD Coated Cutting Tools"

Godfrey C. Onwubolu

Department of Engineering, University of the South Pacific, PO Box 1168, Suva, Fiji

Abstract: An article published in this journal by Sahin and Motorcu[1] developed a surface roughness model based on the response surface method, multiplicative-logarithmically linearized approach for determination of the cutting parameters in turning of AISI 1040 carbon steel. Their published results for the surface roughness show that it appears Sahin and Motorcu[1] have obtained wrong constants (C, m, n, p), thereby resulting in incorrect solutions for the surface roughness prediction model. This note works through the solutions to show how Sahin and Motorcu[1] incorrectly handled the published prediction model constants to their solution. The established predictive model shows that the surface roughness increases with the increase of feed rate but decreases with cutting speed and depth of cut.

Key words: Turning operations, surface roughness, response surface method

INTRODUCTION

The quality of a surface is a significantly important factor in evaluating the productivity of machine tools and machined parts. The surface roughness of machined parts is a significant design specification that is known to have considerable influence on properties such as wear resistance and fatigue strength. It is one of the most important measures in finish cutting (turning, milling, drilling, etc.) operations. Consequently, it is important to achieve a consistent tolerance and surface finish. When the surface finish becomes the main criteria in the quality control department, the productivity of the metal cutting operation is limited by the surface quality. Relatively recent investigations that El-Baradie[2] and Bandyopadhyay[3] conducted have shown that increasing the cutting speed facilitates maximization of productivity and, at the same time, it improves surface quality. According to Gorlenko[4] and Thomas[5], surface finish can be characterized by various parameters. The various roughness height parameters such as average roughness (R_a), smoothening depth (R_p), root mean square (R_q), and maximum peak-to-valley height (R_t) can be closely correlated. The average roughness (R_a) are most widely used in the industry for specifying surface roughness.

Earlier investigators have studied the effect of cutting variables such as speed, feed and depth of cut on surface roughness by taking one variable at a time, which requires the carrying out of many tests in order to be able to draw a conclusion. Optimum cutting conditions are important since they determine to a great extent, the surface quality of the machined parts. However, the response surface methodology (RSM) takes into account the simultaneous variation of the cutting variables and predicts the machining response (the surface roughness). RSM is a statistical method used for analysis is a combination of the design of experiments and regression analysis and statistical inferences. Wu[6] first pioneered the use of response surface methodology in tool life testing. The number of experiments required to develop a surface roughness equation can be reduced markedly as compared to the traditional one-variable-at-a-time approach. Due to the success of RSM, a number of researchers have utilized it to solve the surface roughness prediction problem. Choudhury and El-Baradie[7] utilized RSM for developing surface roughness prediction models for turning operation. Based on response surface methodology, Sahin and Motorcu[1] developed first- and second-order models in their paper for predicting surface roughness.

The published results of Sahin and Motorcu[1] for surface roughness show that it appears that they have obtained wrong constants (*C, m, n, p*), thereby resulting in incorrect solutions for the surface roughness prediction model. This note works through the solutions to show how Sahin and Motorcu[1] incorrectly handled the published prediction model constants to their solution.

Surface roughness model: Sahin and Motorcu[1] represented the relationship between the surface roughness and machining independent variables (speed, feed and depth of cut) by the following:

$$R_a = CV^n f^m d^p \varepsilon \qquad (1)$$

where, R_a is the surface roughness in μm, V, f and d are the cutting speed (m/min), feed (mm/rev) and depth

Corresponding Author: Godfrey C. Onwubolu, Department of Engineering, University of the South Pacific, PO Box 1168, Suva, Fiji

of cut (mm), respectively. *C, n, m, and p* are constants and ε is a random error. In order to facilitate the determination of constants and parameters, the mathematical models were linearized by performing logarithmic transformation[1,7] as follows:

$$\ln R_a = \ln C + n \ln V + m \ln f + p \ln d + \ln \varepsilon \qquad (2)$$

The linear model of Eq. (2) in terms of the estimated response can be written as:

$$\hat{y} = y - \varepsilon = b_o x_o + b_1 x_1 + b_2 x_2 + b_3 x_3 \qquad (3)$$

where $\hat{y}$ is the estimated response of the surface roughness on a logarithmic scale, y is the measured response on a logarithmic scale, $x_o = 1$ (dummy variable), $x_1 = \ln V$, $x_2 = \ln f$, $x_3 = \ln d$, ε is the experimentally random error and the b values are the estimates of the model parameters.

The second order model can be extended from the equation of the first-order model as:

$$\hat{y} = b_o x_o + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_{11} x_1^2 + b_{22} x_2^2$$
$$+ b_{33} x_3^2 + b_{12} x_1 x_2 + b_{13} x_1 x_3 + b_{23} x_2 x_3 \qquad (4)$$

where the b-values are estimated using the method of least squares. The second-order model of Eq. (4) Is useful when the second order effect of *V, f, d* and the two way interaction amongst *V, f* and *d are* significant.

RESULTS

In the present study, the parameters of Eqs. (3) and (4) have been estimated by the method of least squares using a Math-lab computer package.

The publisher of Sahin and Motorcu[1] show that the first-order model for surface roughness (given by their Eq. (8)) is as follows:

$$y = 0.258 - 0.00218x_1 + 0.195x_2 - 0.00453x_3 \qquad (5)$$

This first-order model, which is correct, describing the surface roughness was transformed using their Eq. (6) as:

$$x_1 = \frac{\ln v - \ln(350)}{\ln(402) - \ln(350)}$$

$$x_2 = \frac{\ln f - \ln(0.15)}{\ln(0.172) - \ln(0.15)} \qquad (6)$$

$$x_3 = \frac{\ln t - \ln(0.5)}{\ln(0.575) - \ln(0.5)}$$

Using Eq. (6) (Given by their Eq. (6)), Sahin and Motorcu[1] transformed Eq. (5) (Given by their Eq. (8)) In the following form:

$$R_a = 540.v^{-0.0464} . f^{0.192} . t^{-0.0249} \qquad (7)$$

By substituting for example, the cutting conditions for trial number 1 into Eq. (1), results in R_a=540x (304)$^{-0.0464}$x (0.13)$^{0.169}$x (0.43)$^{-0.0249}$=285.885 µm, which is not comparable at all with neither the average measured value nor the theoretical value in row number 1 of their published Table 3. All other solutions are equally incorrect using their published model. Consequently, the published results of Sahin and Motorcu[1] for surface roughness show that it appears that they have obtained wrong constants *(C, m, n, p)* in their Eq. (8), which is Eq. (7) above, thereby resulting in incorrect solutions for the surface roughness prediction model. This note works through the solutions to show how Sahin and Motorcu[1] incorrectly handled the published prediction model constants to their solution.

A correct model for surface roughness based on published cutting conditions: In this sub-section, step by step analysis is given to postulate the correct model for the surface roughness based on the published cutting conditions of Sahin and Motorcu[1]. Thereafter, some cutting conditions are used to validate the correctness of the model presented in this note. Eq. (6) is further simplified as:

$$x_1 = \frac{\ln V - \ln(350)}{\ln(402) - \ln(350)} = 7.2192 \ln V - 42.289$$

$$x_2 = \frac{\ln f - \ln(0.15)}{\ln(0.172) - \ln(0.15)} = 7.3068 \ln f + 13.8618 \qquad (8)$$

$$x_3 = \frac{\ln t - \ln(0.5)}{\ln(0.575) - \ln(0.5)} = 7.1550 \ln t + 4.9595$$

Substituting these variables into Eq. (5) and noting that $y = \ln R_a$, gives:

$$\ln R_a = 0.258$$
$$- 0.00218 \times (7.2192 \ln v - 42.289) \qquad (9)$$
$$+ 0.19500 \times (7.3068 \ln f + 13.8618)$$
$$- 0.00453 \times (7.1550 \ln t + 4.9595)$$

leading to:

$$\ln R_a = 0.258$$
$$- 0.01573 \ln v + 0.09219$$
$$+ 1.4248 \ln f + 2.70305 \qquad (10)$$
$$- 0.0324 \ln t - 0.022466$$

Table 1 Measured, predicted, published model and correct surface roughness values

Trial No.	R_a, Measured [+]	R_a, Theoretical [+]	R_a, Model [+]	R_a, Present
1	1.104	1.096	285.8855	1. 0626
2	1.080	1.069	282.2029	1.0581
3	1.586	1.5788	301.6731	1.5837
4	1.589	1.555	297.7870	1.5769
5	1.039	1.048	283.8245	1. 0528
6	1.063	1.054	280.1683	1.0483
7	1.576	1.572	299.4982	1.5690
8	1.607	1.598	295.6401	1.5623
9	1.372	1.367	294.6187	1.2995
10	1.334	1.355	287.1174	1.2885
11	1.856	1.857	274.0262	0.8317
12	1.851	1.871	306.7640	1.9220
13	1.281	1.300	292.8343	1.3054
14	1.281	1.277	288.9462	1.2831
15	1.280	1.2721	290.8400	1.2940
16	1.240	1.2721	290.8400	1.2940
17	1.222	1. 2721	290.8400	1.2940
18	1.333	1. 2721	290.8400	1.2940

[+]Sahin and Motorcu[1]

Table 2 Results of measured and predicted values for surface roughness and residual error

Trial No.	R_a, Measured [+]	R_a, Theoretical	LnR_a	LnR_{at}	$LnR_a - LnR_{at}$	$\left(LnR_a - LnR_{at}\right)^2$
1	1.104	1. 0626	0.0989	0.0608	0.0382	0.0015
2	1.080	1.0581	0.0770	0.0565	0.0205	0.0004
3	1.586	1.5837	0.4612	0.4597	0.0015	0.0000
4	1.589	1.5769	0.4631	0.4555	0.0076	0.0001
5	1.039	1. 0528	0.0383	0.0514	-0.0132	0.0002
6	1.063	1.0483	0.0611	0.0472	0.0139	0.0002
7	1.576	1.5690	0.4549	0.4504	0.0045	0.0000
8	1.607	1.5623	0.4744	0.4462	0.0282	0.0008
9	1.372	1.2995	0.3163	0.2620	0.0543	0.0029
10	1.334	1.2885	0.2882	0.2535	0.0347	0.0012
11	1.856	0.8317	-0.1555	-0.1843	0.0288	0.0008
12	1.851	1.9220	0.6157	0.6534	-0.0377	0.0014
13	1.281	1.3054	0.2476	0.2665	-0.0189	0.0004
14	1.281	1.2831	0.2476	0.2493	-0.0017	0.0000
15	1.280	1.2940	0.2469	0.2577	-0.0108	0.0001
16	1.240	1.2940	0.2151	0.2577	-0.0426	0.0018
17	1.222	1.2940	0.2005	0.2577	-0.0572	0.0033
18	1.333	1.2940	0.2874	0.2577	0.0297	0.0009

[+]Sahin and Motorcu[1]

which when tidied up gives the following equations:

$$\ln R_a = 3.03 - 0.01573 \ln v + 1.4248 \ln f - 0.0324 \ln t \qquad (11)$$

$$R_a = \exp\left(3.03 - 0.01573 \ln v + 1.4248 \ln f - 0.0324 \ln t\right) \qquad (12)$$

$$R_a = 20.6754 v^{-0.0153} f^{1.4252} t^{-0.0320} \qquad (13)$$

Substituting the cutting conditions for trial number 1 into Eq. (13) above, of the present note, results in R_a=20. 6751x (304) $^{-0.0153}$x (0.13)$^{1.4252}$x (0.43) $^{-0.0320}$=1.0626 µm, which is comparable with both the average measured value and the theoretical values in row number 1 of Table 3 published in Sahin and Motorcu[1]. Table 1 shows the averaged values, theoretical values, the incorrect model values from the published paper and the theoretical values based on the

correct model presented in this note using Eq. (13). This being the case, we can conclude that Sahin and Motorcu[1] manipulated the coefficients of their surface roughness model incorrectly in their published paper.

Eq. (13) shows that the surface roughness increases with the increase of feed rate but decreases with cutting speed and depth of cut. Table 2 shows that the experimental values are quite close to the predicted values and that the current model constructed in Eq. (13) is able to provide accurate predictions of surface roughness from the cutting process. The sum of squares of the residual of the current model, $J = 0.0160$, while the sum of squares of the deviation from the mean, S=1. 0610 and the coefficient of determination also known as the r-squared value, r^2= 0.9849.

The surface roughness model based on multiplicative-logarithmically linearized approach of Eq. (13) is slightly less efficient than the second-order

traditional response surface model approach, but the advantage is that any value of independent variable could be substituted to determine the response of the surface model. Consequently, in real-life approach, the predictive model based on multiplicative-logarithmically linearized approach has the merits of being used to predict the machining response from a wider range of machining independent variables such as speed, feed and depth of cut.

DISCUSSION AND CONCLUSION

From the analysis carried out based on the published results, we can conclude that Sahin and Motorcu[1] manipulated the coefficients of their surface roughness model incorrectly in their published paper. This therefore raises the question of how they obtained their theoretical values of surface roughness. Consequently, by correctly formulating the surface roughness model, we have identified and rectified the anomaly in the model in Sahin and Motorcu[1].

The current prediction model discussed in this note shows that the surface roughness increases with the increase of feed rate but decreases with cutting speed and depth of cut. Using such a model, it is easy to predict the machining response, which in this case is surface roughness, from a wide range of machining independent variables such as speed, feed and depth of cut outside the range used for experimentation; thereby resulting in a more cost-saving machine operation.

Further research direction includes using an optimization technique to determine the optimal cutting conditions. Including such technique in the prediction model has the additional advantage of finding the best conditions required for the machining independent variables such as speed, feed and depth of cut that would result in the best machining response.

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