

Average Mutual Information Analysis of Multiple Input and Multiple Output System with Multihop Relaying in Wireless Communication System

C. Poongodi and A. Shanmugam
Department of ECE, Bannari Amman Institute of Technology,
Sathyamagalam 638401, India

Abstract: Problem statement: Multiple Input and Multiple Output (MIMO) system with Multihop relaying technique is significant and active areas of wireless communication. In a rich scattering environment MIMO antenna system provides better channel capacity and data rates than single antenna systems. To provide high throughput, reliable transmission and broad coverage, wireless relaying techniques are essential in a variety of applications. In a cellular environment a relay can be used to overcome shadowing effect due to obstacles and multihop relaying can improve the throughput for mobiles suffering from poor signal to interference, noise ratio at the edge of a cell and reduce cell size to increase spectral efficiency. **Approach:** This study analyzes average mutual information of the Ricean channel for single hop Multiple Input Multiple Output (MIMO) system for different antenna configuration and dependence of capacity on the Rice factor for cellular system. The asymptotic capacity of multiuser two-hop MIMO system with Regularization Block Diagonalization (RBD) precoding techniques for Independent Identically Distributed (IID) channel and realistic Mobile to Mobile fading channel model was examined. In this realistic model, Non-Line-Sight (NLOS) propagation conditions are assumed from source mobile station to mobile relay and also from mobile relay to destination station. A non regenerative Amplify and Forward (AF) relay is used to optimize the capacity between the source and destination and also the evaluation was made for multiuser multihop relay system with correlated fading channel model using RBD precoding matrix. **Results:** The simulation results for average mutual information of single hop MIMO relay system for different antenna configuration with ricean channel model, ergodic sum mutual information for two hop relay system with RBD precoding technique and mutual information results for multiuser multihop relay system with correlated channel model was presented. **Conclusion/Recommendations:** Cellular systems generally operated at a fairly low SINR which can be increased on each hop by adding relays. Multiuser two hop relay system with RBD precoding for realistic mobile to mobile fading channel model is simulated and compared with IID channel model. Multiuser Multi-hop relay with optimal precoding and RBD precoding technique results were analyzed for correlated fading channel and found that capacity offered by RBD precoding scheme is better than other relaying systems.

Key words: Amplify-and-forward, mobile to mobile fading channel, multihop relay, correlated channel, regularization block diagonalization, MIMO system, relay network

INTRODUCTION

In MIMO system both transmitter and receiver are provided with more than one antenna. MIMO performs well in scattering rich environment. For rich scattering environment channel it is possible to increase the data rate by transmitting separate information streams on each antenna. MIMO system is a key technique in modern cellular system which provides high spectral efficiency and good coverage. Wireless system must have reasonable throughput with acceptable error rate, but due to fading, multipath propagation, high signal losses and interference, a strongly attenuated and

corrupted signal appears at the receiver. In order to overcome this problem, wireless systems must use sophisticated transmission and receiver processing techniques. In Cellular systems, Signal to Interference and Noise Ratio (SINR) at the mobile user is low when the user is at cell edges. So Multihop relaying and MIMO techniques are used to improve spectral efficiency of cellular system (Jacobson and Krzymien, 2011). MIMO transmission can improve the capacity within a given bandwidth by considering advantage of rich scattering in a typical wireless channel (Foschini and Gans, 1998). MIMO system provides, higher capacity gains at high SINR, but cellular system

Corresponding Author: C. Poongodi, Department of ECE, Bannari Amman Institute of Technology, Sathyamagalam 638401, India

operates at low SINR level leads to poor at cell edges. In multihop relaying (Schultz *et al.*, 2003; Irnich *et al.*, 2003; Bolukbasi *et al.*, 2004) inclusion of transitional wireless relays between transmitter and receiver, to reduce the path loss. SINR can be increased by placing short hop link which reduces the path loss and also avoid the obstacles. This provides to obtain higher link capacities and reliability due to low random signal fluctuation and scattering. This higher SINR level increased the MIMO performance.

In both uplink and downlink transmission multiple antenna elements are used at the base station and terminals to increase the capacity and data rate. Next-generation cellular systems will have large number of users with very high data transmission rates and MIMO is the best tool for increasing spectral efficiency of wireless transmission (Sayadi *et al.*, 2009). The MIMO technologies are mainly used in cellular system due to the existence of spatial diversity and beam forming. The knowledge of Channel State Information (CSI) at base station is essentially needed to improve the throughput of cellular system. Multi-antenna techniques involving multi user scenario delivered the spatial data to different users by utilising all the degrees of freedom in MIMO system. In Single-User MIMO, in addition to beamforming, transmit diversity and spatial multiplexing techniques are also adopted for transmission. This will help to increase the peak user data rate in higher-order MIMO system.

MATERIALS AND METHODS

MIMO with relay system: The MIMO relay scheme is supported both in uplink and downlink of the wireless system. The channel state information is assumed to be known by the receiver. In multiuser system multiple number of users are present at the source side. In the downlink, if a User Equipment (UE) is configured in MU-MIMO transmission mode, it receives the information only about its own precoding matrix. The transmit power level for each user is configured in long term manner to support the higher order modulation like 16QAM and 64 QAM. Zero-Forcing is the most common precoding technique in which the weight vectors are selected as the pseudo-inverse of the channel matrix of the users to avoid interference (Poongodi and Shanmugam, 2011; Caire and Shamai, 2001; Viswanathan *et al.*, 2003). Dirty Paper Coding (Costa, 1993) is another multi-user precoding strategy based on interference pre-subtraction, however the high computational problem occurs when large number of users in the system at the same time. For designing the beamforming vectors, precoding by maximization of signal to leakage ratio

(Tarighat *et al.*, 2003; Wang *et al.*, 2005) is another approach, but it does not have any limitations on the number of transmit antennas and potential for the use of Block Diagonalization (BD).

Relaying is another technique used to improve the performance of wireless system, in terms of coverage and throughput. According to 3GPP (Akyildiz *et al.*, 2010), the use of relay will provide the improvements in data rate, throughput enhancement and coverage extension. The distance between the base station and the UE is separated into distance from the base station to relay and from relay to UE. For minimising the base station, relay and UE transmit power, the relay must be located in the suitable location. The reduction in power consumption is achieved through the lowering the path loss, enhanced relaying schemes and interference control. This reduction in power consumption also lowered the operational costs.

Single hop relay system: Considering single hop relay system with M transmit antennas, N receive antennas and the standard MIMO model described by $M \times N$ matrix H. Elements of matrix H is a random variable, which captures the stochastic nature of wireless channel, consists of both Line of Sight (LOS) and Non-Line of Sight (NLOS) conditions as given below:

$$H = \sqrt{\frac{K_r}{1+K_r}} H_{LOS} + \sqrt{\frac{1}{1+K_r}} H_{NLOS} \quad (1)$$

In Eq. 1, H_{NLOS} is the rayleigh distributed scattered component with unity variance. H_{LOS} is the Line of sight component and its elements are deterministic. H_{LOS} has maximum rank $r_{LOS} = \min(M, N)$ but in practical system H_{LOS} is rank deficient and has rank $r_{LOS} = 1$ (Paulraj *et al.*, 2003; Salo *et al.*, 2006). K_r represents rice factor, defined as the ratio of power in the specular component to the power in the scattered component. The capacity of a MIMO link is given in Eq. 2:

$$C = E \left[\log_2 \left(\det \left(I_N + \rho H H^H / N \right) \right) \right] \quad (2)$$

where, ρ is the Signal to Interference and Noise Ratio (SINR) at the receiver determined by transmit power, path loss and antenna gain and I_N is the identity matrix. The capacity is maximum for full rank channel matrix but H_{LOS} is usually low rank in practical systems. The low rank H_{LOS} and high Rice factor, disintegrate the considerable amount of energy in fewer eigenmodes of H and hence reduced the capacity. Monte Carlo simulation with large number of samples used to find

the average capacity of MIMO system. However, (Salo *et al.*, 2006) provides average mutual information for Ricean MIMO channel $E[IH]$ with rank 1 line of sight component and it is given by Eq. 3:

$$E(I_H) \leq \log_2 \left[1 + \sum_{p=1}^K \sum_{j=0}^1 \left(\frac{pb^2}{M} \right)^p (K_r K_p)^j \times (P-p+1)_{p-j} \binom{K-j}{p-j} \right] \quad (3)$$

where, $K = \min(M, N)$, $b = \sqrt{\frac{1}{K_r + 1}}$, $p = \max(M, N)$ and

$(n)_p$ is the Pochhammer symbol given by $(n)_p = n(n+1) \dots (n+p-1)$ and $n_0 = 1$ (Jacobson and Krzymien, 2011).

In Multihop system, base station transmits data to Mobile Station at the cell edge through Relay Stations. The cell radius (r) is divided into n_{hops} ie., equally spaced relays and r/n_{hops} , $k = 1, 2, \dots, n_{\text{hops}}$. In a MH MIMO system, there are n_{hops} channel matrices, each k hop has M, k transmit antennas and N, k receive antennas. In Eq. 4, the channel matrix for each hop k , is given as:

$$H_k^{n_{\text{hops}}} = \gamma_k^{n_{\text{hops}}} \left[\begin{array}{c} \sqrt{\frac{K_{r,k}^{n_{\text{hops}}}}{1 + K_{r,k}^{n_{\text{hops}}}}} H_{\text{LOS},k}^{n_{\text{hops}}} \\ + \sqrt{\frac{1}{1 + K_{r,k}^{n_{\text{hops}}}}} H_{\text{NLOS},k}^{n_{\text{hops}}} \end{array} \right] \quad (4)$$

Where:

$\gamma_k^{n_{\text{hops}}}$ = Averaged path gain

$K_{r,k}^{n_{\text{hops}}}$ = Rice factor for k^{th} hop. In this channel matrix, Rice factor $K_r(x)$, is represented as:

$$K_r(x) = \begin{cases} 0 & b < x < 5,000\text{m} \\ 10^{1.3-0.003x} & 20\text{m} < x < b \end{cases} \quad (5)$$

From Eq. 5, elements of channel matrix are modeled as Rayleigh random variables when $b < x < 5,000$ m and Ricean (with $K_r > 0$) when $20 \text{ m} < x < b$.

Multuser two hop relay MIMO system: In multuser MIMO system, assumed that number of users (N_u) are present at the source side and i^{th} user has M_i antennas ($i = 1, 2 \dots N_u$). The total number of transmit antennas $M = \sum_{i=1}^{N_u} M_i$ and the channel of i^{th} user is denoted by

$H_i \in C^{M_i \times L}$. The combined channel matrix for all users is given by Eq. 6:

$$H = \begin{bmatrix} H_1^T & H_2^T & \dots & H_{N_u}^T \end{bmatrix}^T \in C^{M \times L} \quad (6)$$

The system model for two hop wireless relay is shown in Fig. 1. The source, relay and destination nodes are equipped with M, N and L antennas respectively. By considering the system having with no line of sight between source and destination due to path loss. The modulated signal vector at the i^{th} user is linearly precoded by precoding matrix. The slot source transmits precoded signal to relay and is given by Eq. 7:

$$y = H_1 F s + n \quad (7)$$

where, y is received data and n is zero mean additive white Gaussian noise at the input of receive antennas. The joint precoding and decoding matrices are denoted by F and G respectively. This MU-MIMO system uses RBD, Regularized Successive Optimization THP (RSO-THP) precoding and Iterative Regularized BD (IRBD) precoding techniques (Young, 2009).

Here an Amplify and Forward (AF) relay is used which simply amplify and forward the received signal to designation and is given by Eq. 8:

$$Y = H_2 x_2 + V_2 \quad (8)$$

where, $x_2 = F_2 y$, H_2 is the $L \times N$ channel matrix between relay and destination, F_2 is relay amplifying matrix and V_2 is complex white Gaussian noise vector with zero mean (Khandaker and Rong, 2010). In this study analysis was made for ergodic capacity of two hop relay system with RBD (Tang and Hua, 2007) precoding techniques for independent identical distributed and Mobile to Mobile fading channel models (Batool and Patzold, 2011).

Multuser multi-hop relay system: The multi-access system with N_u users simultaneously transmitting information to a common destination node through $L-1$ relay node is shown in Fig. 2 (Yue and Xiang, 2011). The $M_i \times 1$ modulated signal vector s_i at the i^{th} user is linearly precoded by the $M_i \times M_i$ user precoding matrix B_i and the precoded signal vector $x_i = B_i s_i$ is transmitted to first relay node (Toding *et al.*, 2010). The signal received at the first relay node is given in Eq. 9:

$$y_j = \sum_{i=1}^{N_u} G_i x_i + v_1 = H_1 x_1 + v_1 \quad (9)$$

where, G_i is $N_1 \times M_1$ MIMO channel matrix between the first relay node and the i^{th} user, v_1 is independent identically distributed additive white Gaussian noise vector at the first relay node, the equivalent first hop MIMO channel $(H_1) = [G_1 \ G_2 \ \dots \ G_{N_u}]$ and $x_1 = F_1 S$ where F_1 is equivalent precoding matrix = $\text{bd}(B_1, B_2 \dots B_{N_u})$.

The input output relationship at the l^{th} relay nodes is given by Eq. 10:

$$x_{j+1} = F_{j+1}y_j, j=1, \dots, L-1 \tag{10}$$

where, F_{j+1} is the amplifying matrix at l^{th} relay node and y_j is signal received at l^{th} relay node written as Eq. 11:

$$y_j = H_j x_j + v_j, j=1, 2, \dots, L-1 \tag{11}$$

where, H_j is the MIMO channel matrix of l^{th} hop. The received signal vector at destination node is given by $y_j = As + \bar{v}$ where A is the equivalent MIMO channel matrix from all users to the destination and it is given by $A = \bigotimes_{i=L}^1 (H_i F_i)$ and assumed that instantaneous Channel State Information (CSI) is available only at the destination node, but CSI is unknown at all users and all relay nodes. In realistic channel MIMO channel is correlated at both transmitter and receiver side. So the instantaneous channel matrices can be represented as Eq. 12:

$$\begin{aligned} G_i &= \phi_r^{1/2} G_i^w \phi_{t,i}^{1/2}, i=1, 2, \dots, N_u \\ H_j &= \phi_{r,j}^{1/2} H_j^w \theta_{t,j}^{1/2}, j=2, \dots, L \end{aligned} \tag{12}$$

Where:

- G_i^w and H_j^w = Gaussian random matrices with IID zero mean and unit variance
- $\theta_{t,j}$ and $\theta_{r,j}$ = Correlation matrix at the transmit and receive side of H_j respectively
- $\phi_{t,i}$ and ϕ_r = Correlation matrices at the transmit and receive side of G_i

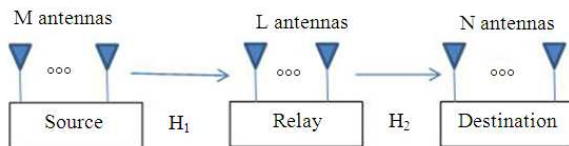


Fig. 1: MIMO AF relay networks with M source, L relay and N destination antennas

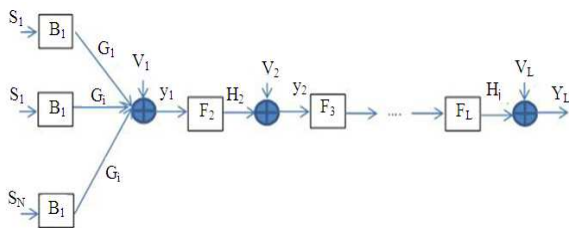


Fig. 2: Block diagram of an N_u users L -hop relay system

The sum mutual information of the users-destination channel is given in Eq. 13:

$$MI = \log \left| I_{N_0} + \bigotimes_{i=1}^L (F_i^H H_i^H) \left(\sum_{j=2}^L \left(\bigotimes_{i=L}^j (H_i F_i \bigotimes_{i=j}^L) \right) + I_{N_L} \right)^{-1} \bigotimes_{i=L}^1 (H_i F_i) \right| \tag{13}$$

where, $| \cdot |$ and $(\cdot)^{-1}$ denote matrix determinant and inversion respectively (Veljko and Martin, 2008). The optimal structures of user precoding matrix (B_i) and relay matrices in the form of singular value decomposition is given by $B_i = U_{\phi,i} \Delta_{b,i}$, $i = 1, \dots, N_u$ and $F_j = U_{\theta,j} \Delta_{f,j} V_{\theta,j-1}^H$, $j = 2, \dots, L$. This result is more general, since it holds for multiuser scenarios by considering MIMO relays.

RESULTS

The addition of multiple relays in single hop MIMO system for Ricean channel model, shorten the hop distance, reduces path loss and scattering. This effect is very much helpful in a single hop link before analysing the entire network. The frequency of operation is 5.8 GHz. The SNR is varied from 0 to 30dB.

The average mutual information for 4×4 and 6×6 MIMO and link with full rank HNLOS and rank 1 HLOS channel is shown in Fig. 3 and 4 respectively. The capacity dependence on Rice factor and antenna configuration is shown in Fig. 5.

Rice factor in cellular systems ranged from 3-20dB, however it is in the steep reduction of capacity level. Figure 6 shows mutual information calculation for multiuser two-hop relay system with RBD precoding matrix for IID and M2M fading channel model.

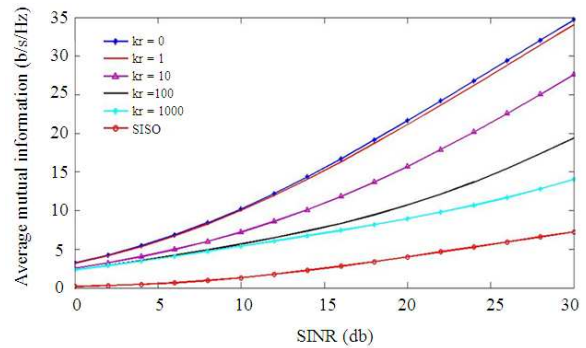


Fig. 3: Average mutual information for (4×4) Ricean MIMO hop

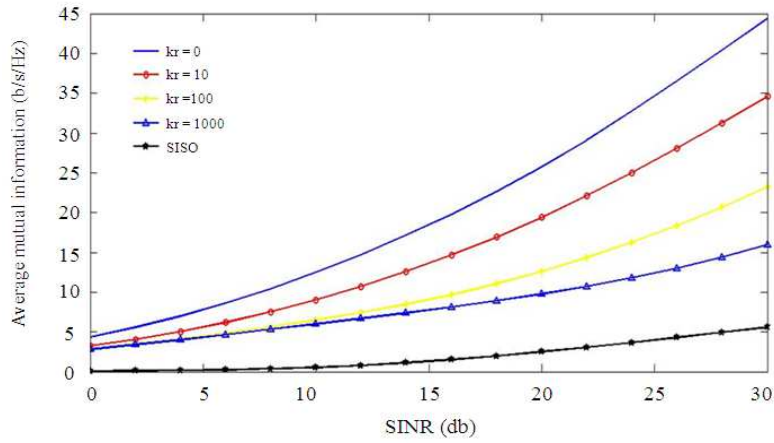


Fig. 4: Average mutual information for (6x6) Ricean MIMO hop

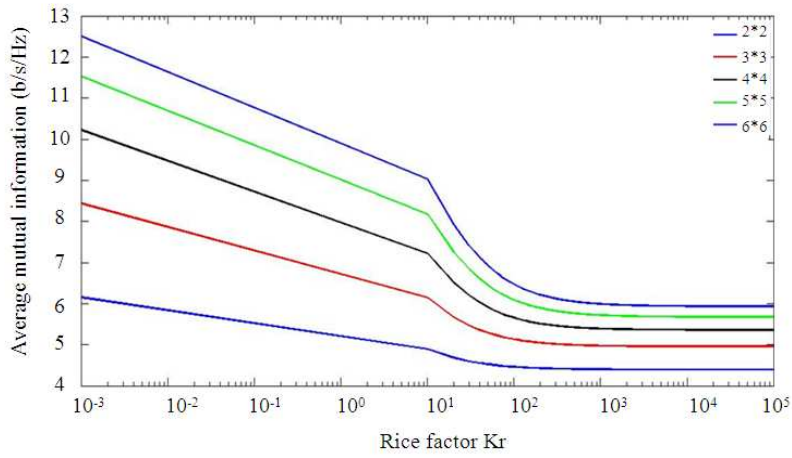


Fig. 5: Average mutual information for a Ricean MIMO hop for SINR = 10dB

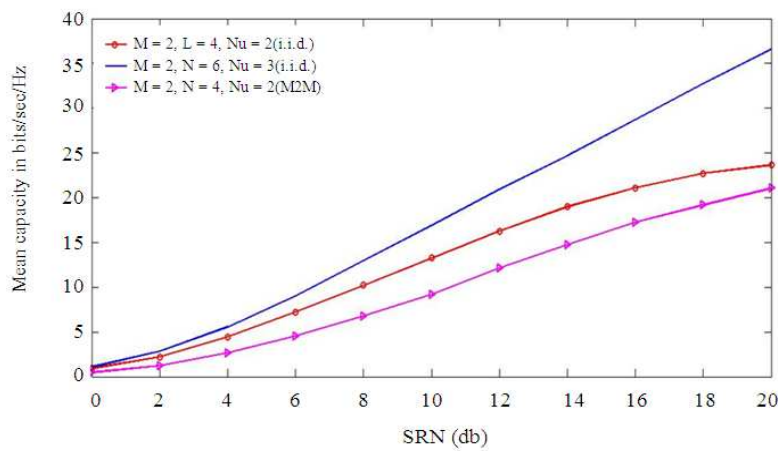


Fig. 6: Mean capacity of two hop relay system for IID and realistic channel model

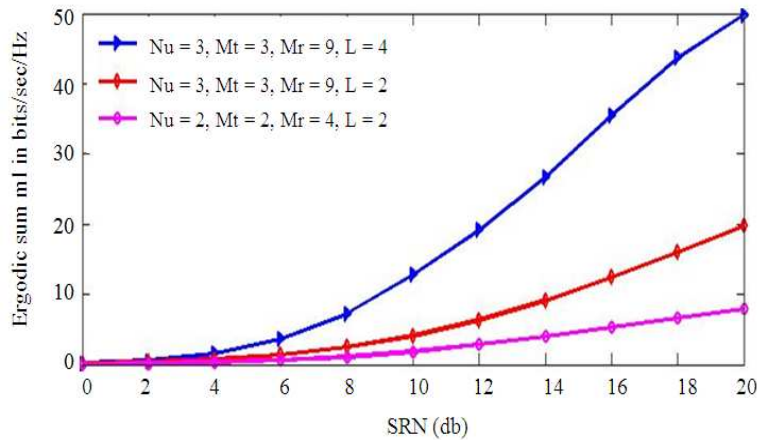


Fig. 7: Ergodic sum mutual information of Multiusermulti-hop relay system with correlated fading model

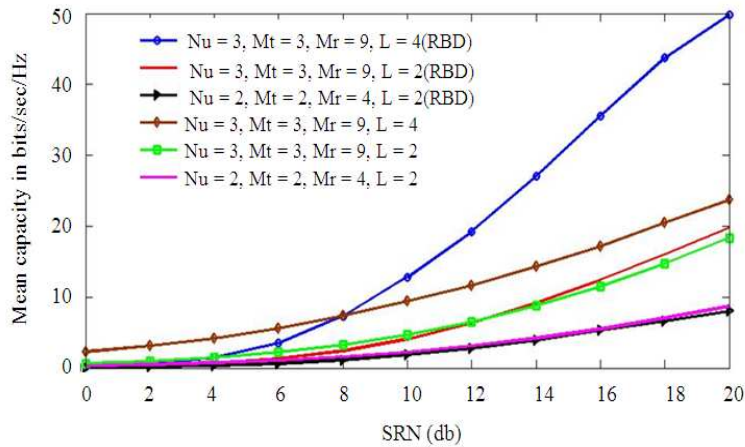


Fig. 8: Mean capacity of multiuser Multihop relay system with optimal and RBD precoding technique

Ergodic sum mutual information of multiuser multi-hop relay system with correlated channel environment is shown in Fig. 7, where relay at each level perform linear precoding on their received signal prior to retransmitting to the next level. Figure 8 shows mutual information for multi-hop relay system with optimal and RBD precoding technique.

DISCUSSION

Cellular systems usually operate at a moderately low SINR as seen in Fig. 3 and 4, however the rate advantage due to MIMO is very less at low SINR level. The SINR increased on every hop by adding relays, but simultaneously the increase in K_r reduces the MIMO capacity gain. From the Eq. 5 indicated that K_r is still around 10 in a distance of 100 m and thus lower the MIMO gain without lost the gain completely. The

dependence of capacity on rice factor and antenna configuration revealed that more antennas offer higher capacities, although the capacity loss occurs with increasing K_r . In two hop and Multihop relay system all N_u users have same number of antennas, all relay nodes and destination nodes are outfitted with same number of antennas (i.e., $N_j = N, j = 1,2,\dots,L$). By assuming that all users having identical transmit power $q_i = P/N_u$, all relay nodes with same transmission power $p_j = P (j = 1,2,\dots,L)$, the mutual information of system increases with number of relay elements.

CONCLUSION

Multiuser two hop relay system with RBD precoding for realistic mobile to mobile fading channel model is simulated and compared with IID channel model by using non-regenerative Amplify and Forward

relay (AF) scheme. Multiuser Multi-hop relay with optimal and RBD precoding technique results are compared with correlated fading channel and found that the capacity obtainable by RBD precoding scheme is better than other relaying systems.

REFERENCES

- Akyildiz, I.F., M.D. Gutierrez-Estevéz and E.C. Reyes 2010. The evolution to 4G cellular systems: LTE-advanced. *Phys. Comm.*, 3: 217-244. DOI: 10.1016/j.phycom.2010.08.001
- Batool, T. and M. Patzold, 2011. A geometrical three-ring-based model for MIMO mobile-to-mobile fading channels in cooperative networks. *EURASIP J. Adv. Sig. Pro.* DOI: 10.1155/2011
- Bolukbasi, H., H Yanikomeroglu, D.D., Falconer and S. Periyalwar 2004. On the capacity of cellular fixed relay networks. *Proceeding of the Canadian Conference on Electrical and Computer Engineering*, May 2-5, IEEE Xplore Press, pp: 2217-2220. DOI: 10.1109/CCECE.2004.1347685
- Caire, G. and S. Shamai 2001. On achievable rates in a multi-antenna Gaussian broadcast channel. *Proceeding of the IEEE International Symposium on Information Theory*, Jun. 24-29, IEEE Xplore Press, Washington, DC., pp: 147-147. DOI: 10.1109/ISIT.2001.936010
- Costa, M. 1993. Writing on dirty paper (corresp.). *IEEE Trans. Inform. Theory*, 29: 439-441 DOI: 10.1109/TIT.1983.1056659
- Foschini, G.J. and M.J. Gans, 1998. On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Pers. Commun.*, 6: 311-335. DOI: 10.1023/A:1008889222784
- Irnich, T., DC Schultz, R Pabst and P. Wienert, 2003. Capacity of a relaying infrastructure for broadband radio coverage of urban areas. *Proceeding of the 58th Vehicular Technology Conference*, Oct. 6-9, IEEE Xplore Press, pp: 2886-2890. DOI: 10.1109/VETECE.2003.1286146
- Jacobson, K.R. and W.A. Krzymien, 2011. Multihop relaying and multiple antenna techniques: performance trade-offs in cellular systems. *Eurasip J. Wireless Comm. Netw.*, 2011: 65-65. DOI: 10.1186/1687-1499-2011-65
- Khandaker, M.R.A. and Y. Rong, 2010. Joint source and relay optimization for multiuser MIMO relay communication systems. *Proceeding of the 4th International Conference on Signal Processing and Communication Systems (ICSPCS)*, Dec.13-15, IEEE Xplore Press, Gold Coast, QLD., pp: 1-6. DOI: 10.1109/ICSPCS.2010.5709701
- Paulraj, A., R. Nabar and D. Gore, 2003. *Introduction to Space-Time Wireless Communications*. 1st Edn., Cambridge University Press, Cambridge, UK., ISBN-10: 0521826152, pp: 277.
- Poongodi, C. and A. Shanmugam, 2011. Ergodic capacity analysis of multiple input multiple output systems with dual hop amplify and forward relay networks. *J. Comput. Sci.*, 7: 1805-1812. DOI: 10.3844/jcssp.2011.1805.1812
- Salo, J., F. Mikas and P. Vainikainen, 2006. An upper bound on the ergodic mutual information in Rician fading MIMO channels. *IEEE Trans Wireless Comm.*, 5: 1415-1421. DOI: 10.1109/TWC.2006.1638662
- Sayadi, F., M. Ismail, N. Misran, K. Jumari and M. Abdullah, 2009. Efficient detection algorithm for a multiple-input and multiple-output multiuser multicarrier code division multiple access in time varying channels. *Am. J. Eng. Applied Sci.*, 2: 635-642. DOI: 10.3844/ajeassp.2009.635.642.
- Schultz, D.C., B. Walke, R. Pabst and T. Irnich 2003. Fixed and planned relay based radio network deployment concepts. *Proceeding of the 10th Wireless World Research Forum (WWRF'03)*, New York, USA., pp: 6-6.
- Tang, X. and Y. Hua, 2007. Optimal design of non-regenerative mimo wireless relays. *IEEE Trans. Wireless Comm.*, 6: 1398-1407. DOI: 10.1109/TWC.2007.348336
- Tarighat, A., M. Sadek and A.H. Sayed, 2003. A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios. *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, Mar. 18-23, IEEE Xplore Press, pp: 1129-1132. DOI: 10.1109/ICASSP.2005.1415913
- Toding, A., Khandaker, M.R.A. and Y. Rong, 2010. Optimal joint source and relay beamforming for parallel MIMO relay networks. *Proceedings of the 6th International Conference on Wireless Communications Networking and Mobile Computing (WiCOM)*, Sept. 23-25, IEEE Xplore Press, Chengdu, pp: 1-4. DOI: 10.1109/WICOM.2010.5600693
- Veljko S. and H. Martin, 2008. Generalized design of multi-user mimo precoding matrices. *IEEE Tran. Wireless comm.*, 7: 953-961: DOI: 10.1109/LCOMM.2008.060709
- Viswanathan, H., S. Venkatesan and H. Huang, 2003. Downlink capacity evaluation of cellular networks with known-interference cancellation. *IEEE J. Sel. Areas Comm.*, 21: 802-811. DOI: 10.1109/JSAC.2003.810346

- Wang, B., J. Zhang and A. Host-Madsen, 2005. On the capacity of MIMO relay channels. *IEEE Trans. Inform. Theory*, 51: 29-43. DOI: 10.1109/TIT.2004.839487
- Young, Y.K., 2009. Capacity of MIMO wireless channel with full-duplex amplify-and-forward relay. *Proceedings of the IEEE 20th International Symposium on Personal Indoor and Mobile Radio Communications*, Sept. 13-16, IEEE Xplore Press, Tokyo, pp: 117-121. DOI: 10.1109/PIMRC.2009.5450313
- Yue, R. and Y. Xiang, 2011. Multiuser multi-hop mimo relay systems with correlated fading channels. *IEEE Trans. Wireless Comm.*, 10: 2835-2840. DOI: 10.1109/TWC.2011.070711.110160