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Review

Some New Gears Aspects

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Abstract: In today's mechanical transmissions most widely used, the gears, are spread across all industries. For this reason, their importance has become overwhelming, which is why we want to recall in this paper some important aspects regarding toothed wheels. It is the geometry, cinematic, the forces and the yields of these mechanisms, which will be presented in the work in the form of newly synthesized relations on modern bases. Another important aspect in toothed wheels is their synthesis by modern methods that avoid tooth interference during operation. To avoid the interference between teeth, we must know the minimum number of teeth of the driving wheel, in function of the pressure angle (normal on the pitch circle, alpha0), in function of the tooth inclination angle (beta) and in function of the transmission ratio (i). In optimal and high-efficiency gearing, gears require a modern design with increased coverage. These achievements can only be achieved today in the context of lowering the value of the alpha engagement angle. Through all the aspects presented, which relate to the dynamics of gears, the work can be considered among those of the optimal dynamic synthesis of the gears.

Keywords: Gears, Gearboxes, Dynamic Synthesis, Yield

Introduction

Gears have spread today in all areas. They have the advantage of working with very high efficiency. In addition, tools can transmit large tasks. Regardless of their size, tools need to be synthesized carefully according to specific conditions.

This paper tries to present the main conditions that must be met for the correct synthesis of a tool.

The beginning of the use of pinion gears should be sought precisely in ancient Egypt at least a thousand years before Christ, where for the first time wheel drive units were used for irrigation and worm gear worm gears for cotton processing.

Then, 230 years BC, in Alexandria, Egypt, the toothed wheel was used again.

These tools have been built and used since ancient times to handle heavy anchors and catapults used on battlefields. These were then introduced into the wind and water mills (as a reduction or multiplication in wind or water pumps) (Fig. 1).

The Antikythera Mechanism is a name given to a complex astronomical device, a 32×16×10 cm device discovered in 1900 in a sunken ship near the coast of Antikythera, an island between Crete and the Greek continent, for which several types of evidence undoubtedly point to around 80 BC. for the date of the shipwreck. The device was made of bronze gears mounted in a wooden box, but due to the fact that it was crushed in the wreck, various parts of the faces were lost and the remainder was then covered with a hard limestone deposit in time at the same time as the corroded metal to a thin core covered with strong metal salts that retains much of the previous bronze shape during the 2000 years of the dive (See Antikythera 1 in Fig. 2).

The modern adventure of the toothed wheel began with the toothed wheel created by Leonardo da Vinci in the fifteenth century. He is also the founder of a new cinema and dynamics, stating, among other things, the principle of overlapping independent movements (Fig. 3).

Benz has created an original toothed and transmission chain engine (patented after 1882, Fig. 4), but the first patent of a toothed gear belongs to British British Starley & Hillman in 1870 (12 years before the Germans) being designed and built to be used for bicycle transmissions and later for motored tricycles.

In Cleveland (USA), begin after 1912 to produce industrial specialized wheels and gears (cylindrical, worm, conical, with straight teeth, inclined or curved; Fig. 5).

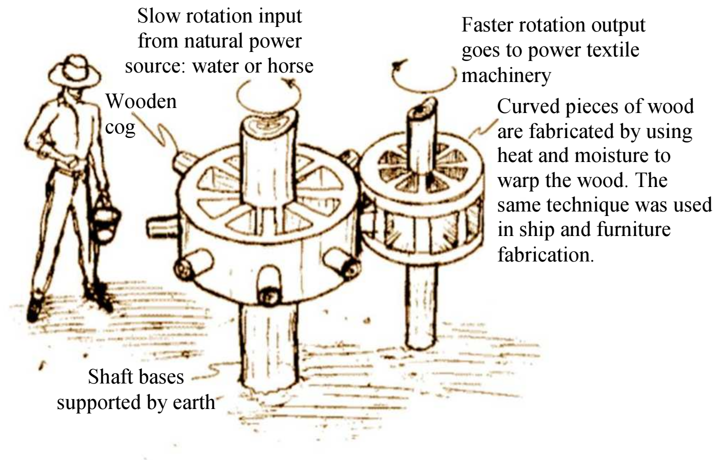

Fig. 1: Transmissions wheeled "spurred" to irrigate crops and worm gears to the cotton processing

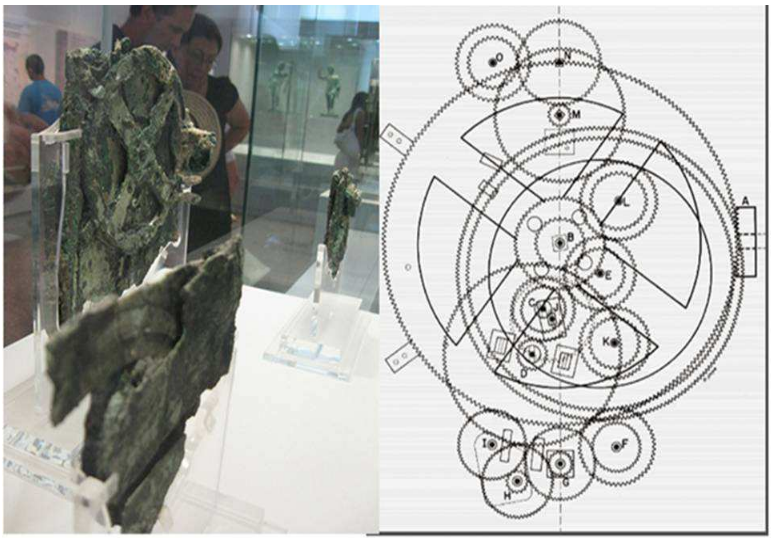

Fig. 2: The Antikythera mechanism is the name given to an astronomical calculating device

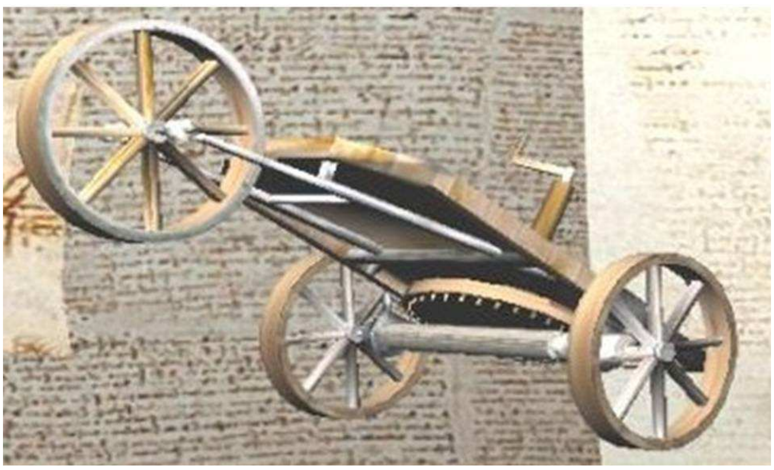

Fig. 3: The modern adventure began with the gear wheel spurred of Leonardo da Vinci, in the fifteenth century

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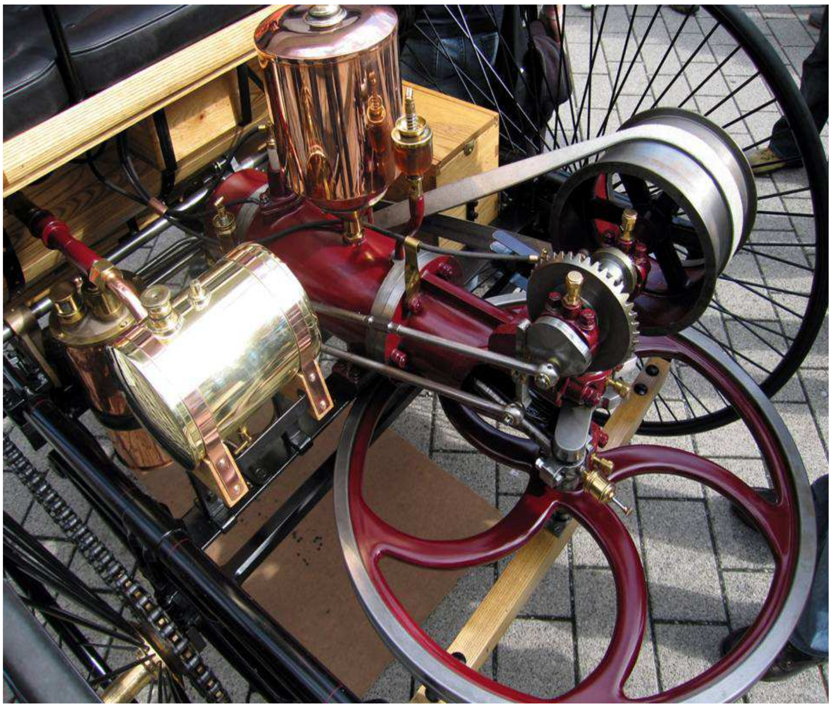

Fig. 4: The Benz patent

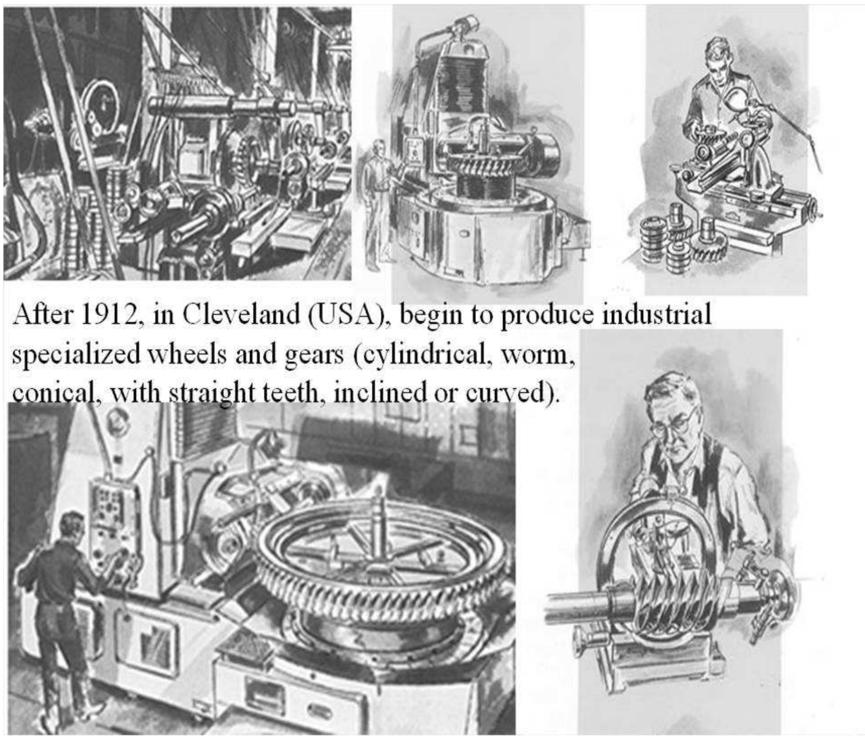

Fig. 5: In Cleveland, after 1912 begin to produce industrial specialized wheels

The gears are present today everywhere, in the mechanical world (In vehicle's industries, in electronics and electro-technique types of equipment, in energetically industries, etc.; Fig. 6).

Fig. 6: Gearings today

The paper presents how to accurately determine the mechanical performance of a gearbox. Based on these relationships, an optimal synthesis of the performance of a classic, mechanical, manual gearshift can be achieved regardless of its operating status (Frățilă *et al*., 2011; Pelecudi, 1967; Antonescu, 2000; Comănescu *et al*., 2010; Aversa *et al*., 2016a; 2016b; 2016c; 2016d; 2017a; 2017b; 2017c; 2017d; 2017e; Mirsayar *et al*., 2017; Cao *et al*., 2013; Dong *et al*., 2013; De Melo *et al*., 2012; Garcia *et al*., 2007; Garcia-Murillo *et al*., 2013; He *et al*., 2013; Lee, 2013; Lin *et al*., 2013; Liu *et al*., 2013; Padula and Perdereau, 2013; Perumaal and Jawahar, 2013; Petrescu and Petrescu, 1995a; 1995b; 1997a; 1997b; 1997c; 2000a; 2000b; 2002a; 2002b; 2003; 2005a; 2005b; 2005c; 2005d; 2005e, 2016a; 2016b; 2016c; 2016d; 2016e; 2013; 2012a; 2012b; 2011; Petrescu *et al*., 2009; 2016a; 2016b; 2016c; 2016d; 2016e; 2017a; 2017b; 2017c; 2017d; 2017e; 2017f; 2017g; 2017h; 2017i; 2017j; 2017k; 2017l; 2017m; 2017n; 2017o; 2017p; 2017q; 2017r; 2017s; 2017t; 2017u; 2017v; 2017w; 2017x; 2017y; 2017z; 2017aa; 2017ab; 2017ac; 2017ad; 2017ae; Petrescu and Calautit, 2016a-b; Reddy *et al*., 2012; Tabaković *et al*., 2013; Tang *et al*., 2013; Tong *et al*., 2013; Wang *et al*., 2013; Wen *et al*., 2012; Antonescu and Petrescu, 1985; 1989; Antonescu *et al*., 1985a; 1985b; 1986; 1987; 1988; 1994; 1997; 2000a; 2000b; 2001; List the first flights, From Wikipedia; Chen and Patton, 1999; Fernandez *et al*., 2005; Fonod *et al*., 2015; Lu *et al*., 2015; 2016; Murray *et al*., 2010; Palumbo *et al*., 2012; Patre and Joshi, 2011; Sevil and Dogan, 2015; Sun and Joshi, 2009; Crickmore, 1997; Donald, 2003; Goodall, 2003; Graham, 2002; Jenkins, 2001; Landis and Dennis, 2005; Clément, Wikipedia; Cayley, Wikipedia; Coandă, Wikipedia; Gunston, 2010; Laming, 2000; Norris, 2010; Goddard, 1916; Kaufman, 1959; Oberth, 1955; Cataldo, 2006; Gruener, 2006; Sherson *et al*., 2006; Williams, 1995; Venkataraman, 1992; Oppenheimer and Volkoff, 1939; Michell, 1784; Droste, 1915; Finkelstein, 1958; Gorder, 2015; Hewish, 1970).

Materials and Methods; Gearings Synthesis

In a cylindrical gearing, forces, speeds, powers and efficiency can be determined using relationships 2.1-2.6 and can be seen in Fig. 7:

$$\begin{cases} F_\tau = F_m \cdot \cos\alpha_1 \\ F_\psi = F_m \cdot \sin\alpha_1 \\ v_2 = v_1 \cdot \cos\alpha_1 \\ v_{12} = v_1 \cdot \sin\alpha_1 \\ \overline{F}_m = \overline{F}_\tau + \overline{F}_\psi \\ \overline{v}_1 = \overline{v}_2 + \overline{v}_{12} \end{cases} \quad (2.1)$$

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Where:

F_m = The motive force (the driving force)
F_τ = The transmitted force (the useful force)
F_ψ = The slide force (the lost force)
v_1 = The velocity of element 1, or the speed of wheel 1 (the driving wheel)
v_2 = The velocity of element 2, or the speed of wheel 2 (the driven wheel)
v_{12} = The relative speed of the wheel 1 in relation with the wheel 2 (this is a sliding speed)

The consumed power (in this case the driving power):

$$P_c \equiv P_m = F_m \cdot v_1 \tag{2.2}$$

The useful power (the transmitted power from the profile 1 to the profile 2) will be written:

$$P_u \equiv P_\tau = F_\tau \cdot v_2 = F_m \cdot v_1 \cdot \cos^2 \alpha_1 \tag{2.3}$$

The lost power will be written:

$$P_\psi = F_\psi \cdot v_{12} = F_m \cdot v_1 \cdot \sin^2 \alpha_1 \tag{2.4}$$

The momentary efficiency of couple will be calculated directly with the next relation:

$$\begin{cases} \eta_i = \dfrac{P_u}{P_c} \equiv \dfrac{P_\tau}{P_m} = \dfrac{F_m \cdot v_1 \cdot \cos^2 \alpha_1}{F_m \cdot v_1} \\ \eta_i = \cos^2 \alpha_1 \end{cases} \tag{2.5}$$

The momentary losing coefficient, will be written:

$$\begin{cases} \psi_i = \dfrac{P_\psi}{P_m} = \dfrac{F_m \cdot v_1 \cdot \sin^2 \alpha_1}{F_m \cdot v_1} = \sin^2 \alpha_1 \\ \eta_i + \psi_i = \cos^2 \alpha_1 + \sin^2 \alpha_1 = 1 \end{cases} \tag{2.6}$$

It can easily see that the sum of the momentary efficiency and the momentary losing coefficient is 1.

Now, one can determine the geometrical elements of gear. These elements will be used in determining the couple efficiency, η.

The main geometric elements belonging to the external cylindrical gear (for straight teeth, beta = 0) can still be determined.

The radius of the basic circle of the wheel 1 (of the driving wheel), (2.7):

$$r_{b1} = \frac{1}{2} \cdot m \cdot z_1 \cdot \cos \alpha_0 \tag{2.7}$$

The radius of the outside circle of wheel 1 (2.8):

$$r_{a1} = \frac{1}{2} \cdot (m \cdot z_1 + 2 \cdot m) = \frac{m}{2} \cdot (z_1 + 2) \tag{2.8}$$

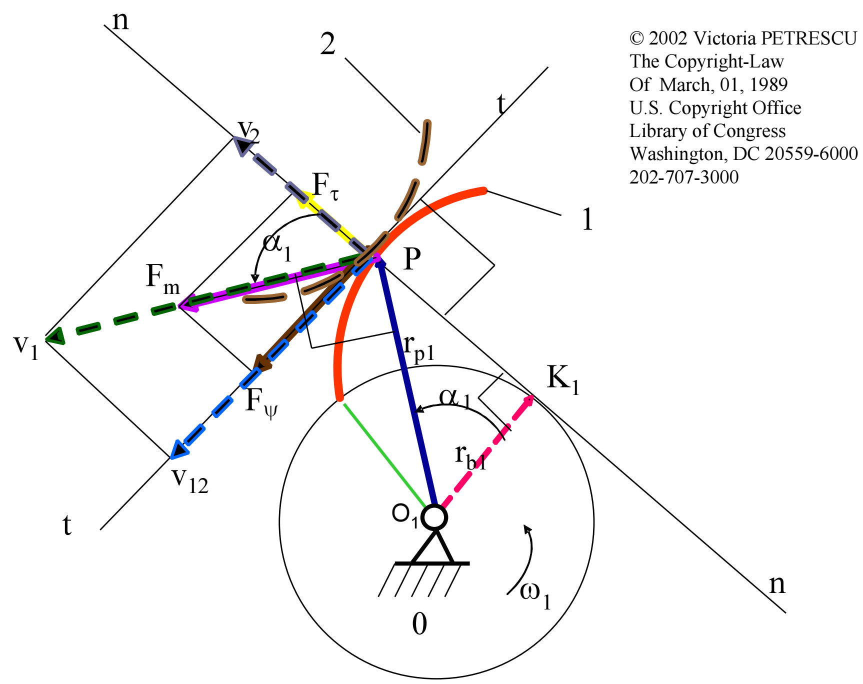

Fig. 7: The forces and the velocities of the gearing

It determines now the maximum pressure angle of the gear (2.9):

$$\cos\alpha_{1M}=\frac{r_{b1}}{r_{a1}}=\frac{\frac{1}{2}\cdot m\cdot z_1\cdot\cos\alpha_0}{\frac{1}{2}\cdot m\cdot(z_1+2)}=\frac{z_1\cdot\cos\alpha_0}{z_1+2} \tag{2.9}$$

And now one determines the same parameters for the wheel 2, the radius of basic circle (2.10) and the radius of the outside circle (2.11) for the wheel 2:

$$r_{b2}=\frac{1}{2}\cdot m\cdot z_2\cdot\cos\alpha_0 \tag{2.10}$$

$$r_{a2}=\frac{m}{2}\cdot(z_2+2) \tag{2.11}$$

Now it can determine the minimum pressure angle of the external gear (2.12, 2.13):

$$\begin{cases} tg\alpha_{1m}=\frac{N}{r_{b1}} \\ N=(r_{b1}+r_{b2})\cdot tg\alpha_0-\sqrt{r_{a2}^2-r_{b2}^2} \\ =\frac{1}{2}\cdot m\cdot(z_1+z_2)\cdot\sin\alpha_0 \\ -\frac{m}{2}\cdot\sqrt{(z_2+2)^2-z_2^2\cdot\cos^2\alpha_0} \\ =\frac{m}{2}\cdot\left[(z_1+z_2)\cdot\sin\alpha_0-\right. \\ \left.\sqrt{z_2^2\cdot\sin^2\alpha_0+4\cdot z_2+4}\right] \end{cases} \tag{2.12}$$

$$tg\alpha_{1m}=\left[(z_1+z_2)\cdot\sin\alpha_0\right. \\ \left.\sqrt{z_2^2\cdot\sin^2\alpha_0+4\cdot z_2+4}\right]/(z_1\cdot\cos\alpha_0) \tag{2.13}$$

Now we can determine, for the external gear, the minimum (2.13) and the maximum (2.9) pressure angle for the right teeth. For the external gear with bended teeth ($\beta\neq 0$) it uses the relations (2.14, 2.15 and 2.16):

$$tg\alpha_t=\frac{tg\alpha_0}{\cos\beta} \tag{2.14}$$

$$tg\alpha_{1m}=\left[(z_1+z_2)\cdot\frac{\sin\alpha_t}{\cos\beta}\right. \\ \left.\sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta}+4\cdot\frac{z_2}{\cos\beta}+4}\right]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{2.15}$$

$$\cos\alpha_{1M}=\frac{\frac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\frac{z_1}{\cos\beta}+2} \tag{2.16}$$

For the internal gear with bended teeth ($\beta\neq 0$) it uses the relations (2.14 with 2.17, 2.18-A, or with 2.19, 2.20-B):

A. When the Driving Wheel 1, Has External Teeth:

$$tg\alpha_{1m}=\left[(z_1-z_2)\cdot\frac{\sin\alpha_t}{\cos\beta}\right. \\ \left.+\sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta}-4\cdot\frac{z_2}{\cos\beta}+4}\right]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{2.17}$$

$$\cos\alpha_{1M}=\frac{\frac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\frac{z_1}{\cos\beta}+2} \tag{2.18}$$

B. When the Driving Wheel 1, Have Internal Teeth:

$$tg\alpha_{1M}=\left[(z_1-z_2)\cdot\frac{\sin\alpha_t}{\cos\beta}\right. \\ \left.+\sqrt{z_2^2\cdot\frac{\sin^2\alpha_t}{\cos^2\beta}+4\cdot\frac{z_2}{\cos\beta}+4}\right]\cdot\frac{\cos\beta}{z_1\cdot\cos\alpha_t} \tag{2.19}$$

$$\cos\alpha_{1m}=\frac{\frac{z_1\cdot\cos\alpha_t}{\cos\beta}}{\frac{z_1}{\cos\beta}-2} \tag{2.20}$$

The mechanical efficiency of the cylindrical gear shall be determined by integrating the instantaneous efficiency across all gear sections of the gear unit starting from the minimum pressure angle and going up to the maximum pressure angle as defined in expression (2.21):

$$\begin{aligned}\eta&=\frac{1}{\Delta\alpha}\cdot\int_{\alpha_m}^{\alpha_M}\eta_i\cdot d\alpha=\frac{1}{\Delta\alpha}\int_{\alpha_m}^{\alpha_M}\cos^2\alpha\cdot d\alpha \\ &=\frac{1}{2\cdot\Delta\alpha}\cdot\left[\frac{1}{2}\cdot\sin(2\cdot\alpha)+\alpha\right]_{\alpha_m}^{\alpha_M} \\ &=\frac{1}{2\cdot\Delta\alpha}\left[\frac{\sin(2\alpha_M)-\sin(2\alpha_m)}{2}+\Delta\alpha\right] \\ &=\frac{\sin(2\cdot\alpha_M)-\sin(2\cdot\alpha_m)}{4\cdot(\alpha_M-\alpha_m)}+0.5\end{aligned} \tag{2.21}$$

External gears are the most common, not because they are the best, but because they are easier to design and build (Fig. 8a). Internal gears can be much more efficient and more reliable if and only if properly designed (Fig. 8b). At inner engagement the teeth in contact make better contact not only on a point or line as on the outside but on a curve or surface, contact being larger, more natural, stronger, more complete and without wear, noises, shocks, like in external gears. However, due to the fact that the internal gears are provided with additional conditions for avoiding the teeth interference in contact, the correct design is much more difficult and from a technological point of view it sometimes does not work correctly, leading to slight random interferences during the operation of the built-in gear, which in the course of time lead to premature wear, noises, or even blocking in operation, although their operation should have been much superior theoretically and for this reason the design difficulties most often give up the superiority internal gears preferring the choice of the outer ones.

To an external gearing, contact between profiles shall only be made to a single point, while at the internal gearing the contact between profiles is by winding each other (Fig. 9).

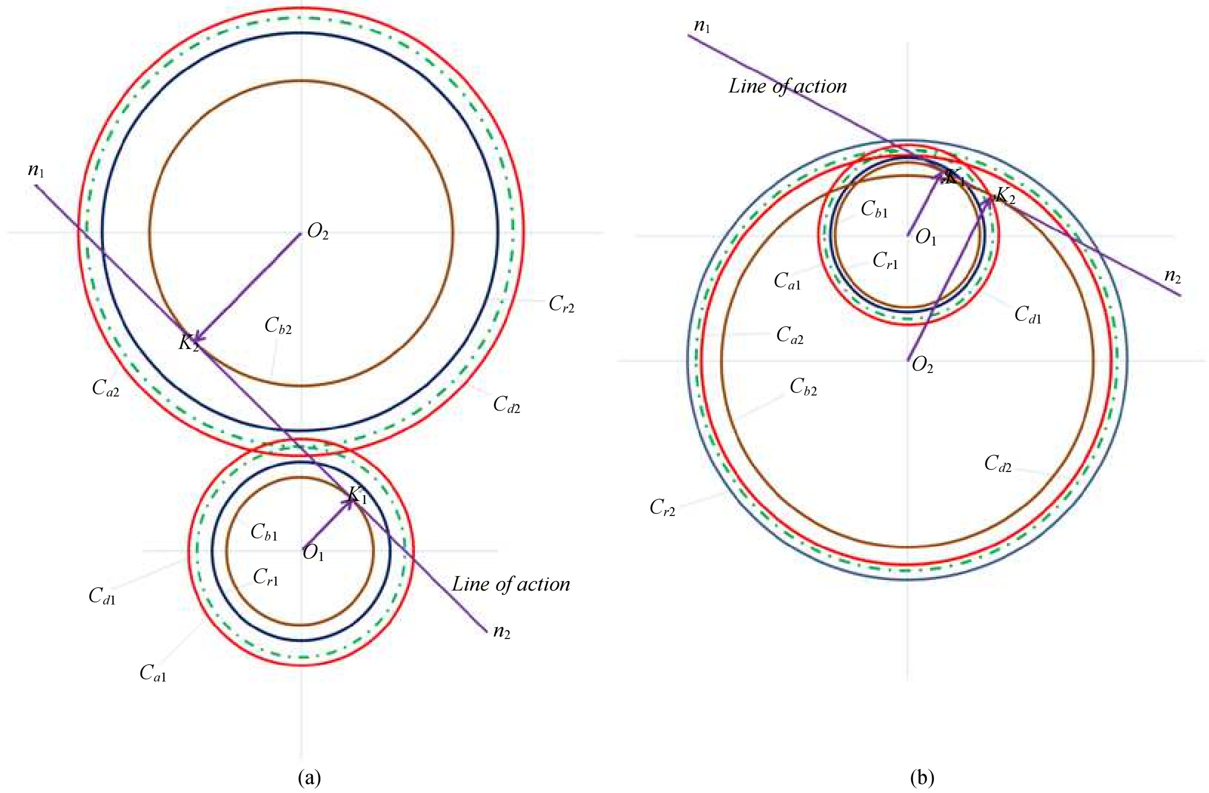

Fig. 8: (a) An external gearing; (b) An internal gearing

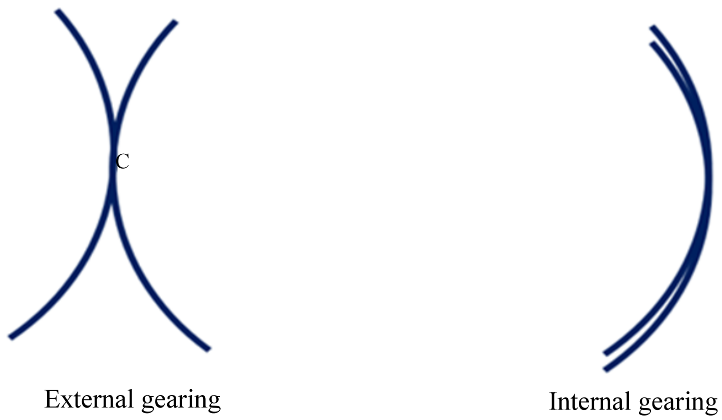

Fig. 9: Contact between profiles

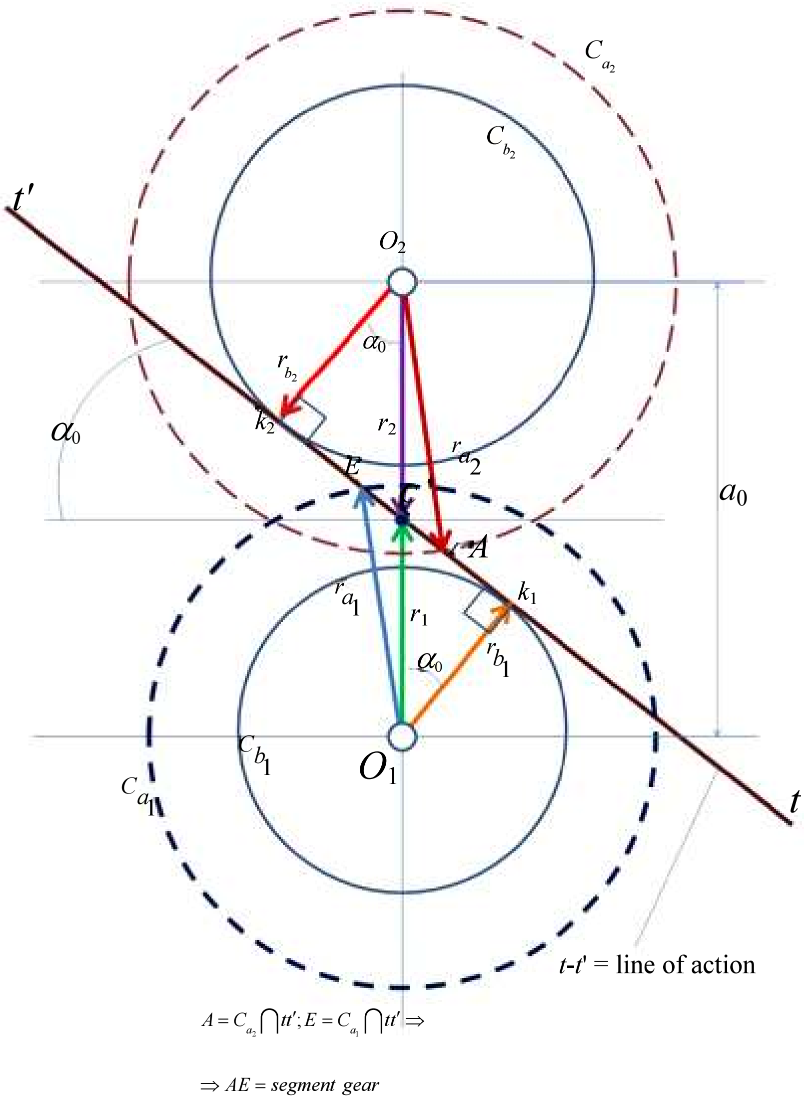

Fig. 10: Line of action (*t-t*') at an external gearing

Results; Gears Synthesis by Avoid the Interferences

In order to avoid interference phenomenon, point A must lie between C and K_1 (the addendum circle of the wheel 2, C_{a2} need to cut the line of action between points C and K_1 and under no circumstances does not exceed the point K_1). Similarly, C_{a1} addendum circle must cut the action line between points C and K_2, resulting in point E, which in no circumstances, does not exceed the point K_2.

The conditions to avoid the phenomenon of interference can be written with the relations (3.1).

The basic conditions of interference, are the same (CA<K1C; CE<K2C), but the originality of this new presented method consist in the mode in which it was solved the classical relationship (see the system 3.1) (Fig. 10).

The system (3.3) represents a simple, unitary and general relationship capable of generating functional solutions for gears, giving the minimum number of teeth of wheel 1 (motor wheel) to avoid interference. In the appendix Table 1-15 an alpha0 value (35°) will be chosen and the beta angles (from 0° to 40°) and the transmission ratio i (from 1 to 80) are incrementally incremented in order to thus getting the minimum number of teeth correctly.

Then, the alpha value (from 35° to 5°) will be decreased successively.

At the internal gearbox, the interference avoidance condition is the same as for the external gear (relationship 3.3):

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$$\begin{cases} CA < K_1C \quad and \quad CE < K_2C \\ CA = K_2A - K_2C = \sqrt{r_{a_2}^2 - r_{b_2}^2} - r_2 \cdot \sin\alpha_0;\ \ CA < K_1C \\ \Rightarrow \sqrt{r_{a_2}^2 - r_{b_2}^2} - r_2 \cdot \sin\alpha_0 < r_1 \cdot \sin\alpha_0 \Rightarrow \sqrt{r_{a_2}^2 - r_{b_2}^2} < (r_1 + r_2) \cdot \sin\alpha_0 \\ \Rightarrow d_{a_2}^2 - d_{b_2}^2 < (d_1 + d_2)^2 \cdot \sin^2\alpha_0 \\ \Rightarrow m^2 \cdot (z_2 + 2)^2 - m^2 \cdot z_2^2 \cdot \cos^2\alpha_0 < m^2 \cdot (z_1 + z_2)^2 \cdot \sin^2\alpha_0 \\ \Rightarrow z_2^2 + 4 \cdot z_2 + 4 - z_2^2 < z_1^2 \cdot \sin^2\alpha_0 + 2 \cdot z_1 \cdot z_2 \cdot \sin^2\alpha_0 \\ \Rightarrow 4 \cdot z_2 + 4 < z_1^2 \cdot \sin^2\alpha_0 + 2 \cdot z_1 \cdot z_2 \cdot \sin^2\alpha_0 \\ from \quad CE < K_2C \Rightarrow 4 \cdot z_1 + 4 < z_2^2 \cdot \sin^2\alpha_0 + 2 \cdot z_1 \cdot z_2 \cdot \sin^2\alpha_0 \\ it \ obtains \ the \ system \begin{cases} 4 \cdot z_2 + 4 < z_1^2 \cdot \sin^2\alpha_0 + 2 \cdot z_1 \cdot z_2 \cdot \sin^2\alpha_0 \\ 4 \cdot z_1 + 4 < z_2^2 \cdot \sin^2\alpha_0 + 2 \cdot z_1 \cdot z_2 \cdot \sin^2\alpha_0 \end{cases} \\ take \quad i \equiv |i_{12}| = \dfrac{z_2}{z_1} \Rightarrow z_2 = i \cdot z_1; result \ \ the \ \ system \\ \begin{cases} \sin^2\alpha_0 \cdot (1 + 2 \cdot i) \cdot z_1^2 - 2 \cdot 2 \cdot i \cdot z_1 - 4 > 0 \\ \sin^2\alpha_0 \cdot (i^2 + 2 \cdot i) \cdot z_1^2 - 2 \cdot 2 \cdot z_1 - 4 > 0 \end{cases} with \ \ the \ \ solutions: \\ \begin{cases} z_{1_{1,2}} = \dfrac{2 \cdot i \pm 2 \cdot \sqrt{i^2 + \sin^2\alpha_0 + 2 \cdot i \cdot \sin^2\alpha_0}}{(2 \cdot i + 1) \cdot \sin^2\alpha_0} \\ z_{1_{3,4}} = \dfrac{2 \pm 2 \cdot \sqrt{1 + i^2 \cdot \sin^2\alpha_0 + 2 \cdot i \cdot \sin^2\alpha_0}}{(2 \cdot i + i^2) \cdot \sin^2\alpha_0} \end{cases} it \ \ keeps \ \ solutions + \\ \begin{cases} z_{1_2} = 2 \cdot \dfrac{i + \sqrt{i^2 + \sin^2\alpha_0 + 2 \cdot i \cdot \sin^2\alpha_0}}{(2 \cdot i + 1) \cdot \sin^2\alpha_0} \\ z_{1_4} = 2 \cdot \dfrac{1 + \sqrt{1 + i^2 \cdot \sin^2\alpha_0 + 2 \cdot i \cdot \sin^2\alpha_0}}{(2 \cdot i + i^2) \cdot \sin^2\alpha_0} \end{cases} \end{cases} \tag{3.1}$$

Relationship which generates z_{1_4} always gives lower values than the relationship which generates z_{1_2} so it is sufficient the condition (3.2) for finding the minimum number of teeth of the wheel 1, necessary to avoid interference:

$$z_{\min} \equiv z_{1_2} = 2 \cdot \frac{i + \sqrt{i^2 + \sin^2\alpha_0 + 2 \cdot i \cdot \sin^2\alpha_0}}{(2 \cdot i + 1) \cdot \sin^2\alpha_0} \tag{3.2}$$

When we have inclined teeth, one takes $z_{min} \rightarrow z_{min}/\cos\beta$ and $\alpha_0 \rightarrow \alpha_{0t}$ and the relationship (3.2) takes the form (3.3). The minimum number of teeth of the driving wheel 1, is a function on some parameters: The pressure angle (normal on the pitch circle, α_0), the tooth inclination angle (β) and the transmission ratio ($i = |i_{12}| = |-z_2/z_1| = z_2/z_1$), (see the relationship 3.3):

$$\begin{cases} z_{\min} \equiv z_{1_2} = 2 \cdot \cos\beta \cdot \dfrac{i + \sqrt{i^2 + \sin^2\alpha_{0t} + 2 \cdot i \cdot \sin^2\alpha_{0t}}}{(2 \cdot i + 1) \cdot \sin^2\alpha_{0t}} \\ where: tg\alpha_{0t} = \dfrac{tg\alpha_0}{\cos\beta} \Rightarrow \alpha_{0t} = arctg\left(\dfrac{tg\alpha_0}{\cos\beta}\right) \end{cases} \tag{3.3}$$

In addition, the inner gear can also write the additional condition of the wheel with internal teeth (systems 3.4 and 3.5). If the mechanism is designed and built without checking these two additional conditions for the existence of an internal gear, it will not work properly. As it has already shown, the inner gear is much superior in operation to the outside, but only when rigorous design and construction, its manufacturing technology being much more difficult than that of the classic outer gear:

$$\begin{cases} r_{a_2} > r_{b_2} \Rightarrow \frac{m}{2} \cdot \left(\frac{z_2}{\cos\beta} - 2 \right) > \frac{m}{2} \cdot \frac{z_2}{\cos\beta} \cdot \cos\alpha_{0t} \\ \Rightarrow \frac{z_2}{\cos\beta} - 2 > \frac{z_2}{\cos\beta} \cdot \cos\alpha_{0t} \Rightarrow z_2 > \frac{2 \cdot \cos\beta}{1 - \cos\alpha_{0t}} \end{cases} \quad (3.4)$$

$$\begin{cases} z_2 > \frac{2 \cdot \cos\beta}{1 - \cos\alpha_{0t}} \\ \cos\alpha_{0t} = \frac{1}{\sqrt{1 + tg^2\alpha_{0t}}} = \frac{1}{\sqrt{1 + \frac{tg^2\alpha_0}{\cos^2\beta}}} = \frac{\cos\beta}{\sqrt{\cos^2\beta + tg^2\alpha_0}} \\ \Rightarrow z_2 > \frac{2 \cdot \cos\beta}{1 - \frac{\cos\beta}{\sqrt{\cos^2\beta + tg^2\alpha_0}}} \Rightarrow z_2 > \frac{2 \cdot \cos\beta \cdot \sqrt{\cos^2\beta + tg^2\alpha_0}}{\sqrt{\cos^2\beta + tg^2\alpha_0} - \cos\beta} \\ \Rightarrow z_2 > \frac{2 \cdot \cos\beta \cdot \sqrt{\cos^2\beta + tg^2\alpha_0} \cdot \left(\sqrt{\cos^2\beta + tg^2\alpha_0} + \cos\beta \right)}{tg^2\alpha_0} \\ \Rightarrow z_2 > \frac{2 \cdot \cos\beta}{tg^2\alpha_0} \cdot \left(\cos^2\beta + tg^2\alpha_0 + \cos\beta \cdot \sqrt{\cos^2\beta + tg^2\alpha_0} \right) \\ \Rightarrow z_2 > \frac{2 \cdot \cos^3\beta}{tg^2\alpha_0} \cdot \left(1 + \frac{tg^2\alpha_0}{\cos^2\beta} + \sqrt{1 + \frac{tg^2\alpha_0}{\cos^2\beta}} \right) \end{cases} \quad (3.5)$$

It should also be mentioned that additional relations (3.6) have also been used.

$$\begin{cases} r_1 = \frac{1}{2} m \cdot z_1; \; r_2 = \frac{1}{2} m \cdot z_2; \; r_{b_1} \\ = \frac{1}{2} m \cdot z_1 \cdot \cos\alpha_0; \; r_{b_2} = \frac{1}{2} m \cdot z_2 \cdot \cos\alpha_0 \\ r_{a_1} = r_1 + m = \frac{1}{2} m \cdot z_1 + \frac{2}{2} m = \frac{m}{2} \cdot (z_1 + 2) \\ r_{a_2} = r_2 - m = \frac{1}{2} m \cdot z_2 - \frac{2}{2} m = \frac{m}{2} \cdot (z_2 - 2) \\ r_{r_1} = r_1 - 1.25m = \frac{1}{2} m \cdot z_1 - \frac{2.5}{2} m = \frac{m}{2} \cdot (z_1 - 2.5) \\ r_{r_2} = r_2 + 1.25m = \frac{1}{2} m \cdot z_2 + \frac{2.5}{2} m = \frac{m}{2} \cdot (z_2 + 2.5) \end{cases} \quad (3.6)$$

Discussion; Determining the Gearing Performance Depending on the Degree of Coverage

In this section, there is briefly presented a completely original method of determining the efficiency of parallel gear gears. Based on the computational relationships presented, the dynamic synthesis of the gears can be made so as to result in mechanisms with high efficiency in operation.

The originality of the method consists in determining the yield (which does not take into account the friction coefficient in the coupling, this being considered only an additional effect and not the main cause that produces the effective mechanical efficiency, the mechanical efficiency of a machine depends on the authors' mainly by the transmission angle of the main coupler of the mechanism).

Calculate the yield of a gear with a fixed spindle gear, considering that at a certain moment there are several pairs of drive drums, not just one.

It starts from the idea of having four pairs of drums in engagement (simultaneous). The first pair of teeth (which go on the right-to-left engagement line as it engages) are the engagement point *i*, defined by the radius of the r_{i1} and the angle (pressure) of the position a_{i1}; the forces at this point are the force of the F_{mi} motors, perpendicular to the point and position of the vector and the force transmitted from the wheel 1 to the second wheel by the point *i*, F_{ti}, parallel to the engagement line and pointing from the wheel 1 to the wheel 2, the transmission force being basically the projection of the drive force on the engagement axis (line); the defined speeds are similar to the forces (for the original cinematic, precision); the same parameters will also be defined for the other three points of engagement, *j*, *k*, *l* (following the drawing in Fig. 11).

Write the relationships between speeds (4.1) first:

$$\begin{aligned} v_{\tau i} &= v_{mi} \cdot \cos\alpha_i = r_i \cdot \omega_1 \cdot \cos\alpha_i = r_{b1} \cdot \omega_1 \\ v_{\tau j} &= v_{mj} \cdot \cos\alpha_j = r_j \cdot \omega_1 \cdot \cos\alpha_j = r_{b1} \cdot \omega_1 \\ v_{\tau k} &= v_{mk} \cdot \cos\alpha_k = r_k \cdot \omega_1 \cdot \cos\alpha_k = r_{b1} \cdot \omega_1 \\ v_{\tau l} &= v_{ml} \cdot \cos\alpha_l = r_l \cdot \omega_1 \cdot \cos\alpha_l = r_{b1} \cdot \omega_1 \end{aligned} \quad (4.1)$$

From relations (4.1) one obtains the equality of tangential speeds (4.2) and we express the motor speeds (4.3):

$$v_{\tau i} = v_{\tau j} = v_{\tau k} = v_{\tau l} = r_{b1} \cdot \omega_1 \quad (4.2)$$

$$v_{mi} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_i}; v_{mj} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_j}; v_{mk} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_k}; v_{ml} = \frac{r_{b1} \cdot \omega_1}{\cos\alpha_l} \quad (4.3)$$

The forces simultaneously transmitted at the four points must be equal to each other (4.4):

$$F_{\tau i} = F_{\tau j} = F_{\tau k} = F_{\tau l} = F_\tau \quad (4.4)$$

Engine forces shall be deducted (4.5):

$$F_{mi} = \frac{F_\tau}{\cos\alpha_i}; F_{mj} = \frac{F_\tau}{\cos\alpha_j}; F_{mk} = \frac{F_\tau}{\cos\alpha_k}; F_{ml} = \frac{F_\tau}{\cos\alpha_l} \quad (4.5)$$

Instant yield is written as (4.6):

$$\begin{aligned}\eta_i &= \frac{P_u}{P_c} = \frac{P_\tau}{P_m} = \frac{F_{\tau i}\cdot v_{\tau i} + F_{\tau j}\cdot v_{\tau j} + F_{\tau k}\cdot v_{\tau k} + F_{\tau l}\cdot v_{\tau l}}{F_{mi}\cdot v_{mi} + F_{mj}\cdot v_{mj} + F_{mk}\cdot v_{mk} + F_{ml}\cdot v_{ml}} \\ &= \frac{4\cdot F_\tau\cdot r_{b1}\cdot\omega_1}{\frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_i} + \frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_j} + \frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_k} + \frac{F_\tau\cdot r_{b1}\cdot\omega_1}{\cos^2\alpha_l}} \\ &= \frac{4}{\frac{1}{\cos^2\alpha_i} + \frac{1}{\cos^2\alpha_j} + \frac{1}{\cos^2\alpha_k} + \frac{1}{\cos^2\alpha_l}} \\ &= \frac{4}{4 + tg^2\alpha_i + tg^2\alpha_j + tg^2\alpha_k + tg^2\alpha_l}\end{aligned} \tag{4.6}$$

The help lines used shall be marked with (4.7) and (4.8):

$$\begin{cases} K_1 i = r_{b1}\cdot tg\alpha_i; K_1 j = r_{b1}\cdot tg\alpha_j; \\ K_1 k = r_{b1}\cdot tg\alpha_k; K_1 l = r_{b1}\cdot tg\alpha_l \\ K_1 j - K_1 i = r_{b1}\cdot\left(tg\alpha_j - tg\alpha_i\right); \\ K_1 j - K_1 i = r_{b1}\cdot\frac{2\cdot\pi}{z_1} \Rightarrow tg\alpha_j = tg\alpha_i + \frac{2\cdot\pi}{z_1} \\ K_1 k - K_1 i = r_{b1}\cdot(tg\alpha_k - tg\alpha_i); \\ K_1 k - K_1 i = r_{b1}\cdot 2\cdot\frac{2\cdot\pi}{z_1} \Rightarrow tg\alpha_k = tg\alpha_i + 2\cdot\frac{2\cdot\pi}{z_1} \\ K_1 l - K_1 i = r_{b1}\cdot\left(tg\alpha_l - tg\alpha_i\right); \\ K_1 l - K_1 i = r_{b1}\cdot 3\cdot\frac{2\cdot\pi}{z_1} \Rightarrow tg\alpha_l = tg\alpha_i + 3\cdot\frac{2\cdot\pi}{z_1} \end{cases} \tag{4.7}$$

$$\begin{aligned} tg\alpha_j &= tg\alpha_i \pm \frac{2\cdot\pi}{z_1}; tg\alpha_k \\ &= tg\alpha_i \pm 2\cdot\frac{2\cdot\pi}{z_1}; tg\alpha_l = tg\alpha_i \pm 3\cdot\frac{2\cdot\pi}{z_1} \end{aligned} \tag{4.8}$$

Relationships (4.8) were retained where the plus sign is for the gears to which the driving wheel 1 has external gear (external or internal gearing) and the minus sign is used when the driving wheel 1 has internal gearing, i.e., when the driving wheel 1 is a ring (only at the inner engagement). The instantaneous yield in the expression (4.6) uses relations (4.8) and takes the form (4.9):

$$\begin{cases} \eta_i = \frac{4}{4 + tg^2\alpha_i + tg^2\alpha_j + tg^2\alpha_k + tg^2\alpha_l} \\ = \frac{4}{4 + tg^2\alpha_i + \left(tg\alpha_i \pm \frac{2\pi}{z_1}\right)^2 + \left(tg\alpha_i \pm 2\cdot\frac{2\pi}{z_1}\right)^2 + (tg\alpha_i \pm 3\cdot\frac{2\pi}{z_1})^2} \\ = \frac{4}{4 + 4\cdot tg^2\alpha_i + \frac{4\pi^2}{z_1^2}\cdot\left(0^2+1^2+2^2+3^2\right) \pm 2\cdot tg\alpha_i\cdot\frac{2\pi}{z_1}\cdot\left(0+1+2+3\right)} \\ = \frac{1}{1 + tg^2\alpha_i + \frac{4\pi^2}{E\cdot z_1^2}\cdot\sum_{i=1}^{E}\left(i-1\right)^2 \pm 2\cdot tg\alpha_i\cdot\frac{2\pi}{E\cdot z_1}\cdot\sum_{i=1}^{E}\left(i-1\right)} \\ = \frac{1}{1 + tg^2\alpha_1 + \frac{4\pi^2}{E\cdot z_1^2}\cdot\frac{E\cdot(E-1)\cdot(2\cdot E-1)}{6} \pm \frac{4\pi\cdot tg\alpha_1}{E\cdot z_1}\cdot\frac{E\cdot(E-1)}{2}} \\ = \frac{1}{1 + tg^2\alpha_1 + \frac{2\pi^2\cdot\left(E-1\right)\cdot\left(2E-1\right)}{3\cdot z_1^2} \pm \frac{2\pi\cdot tg\alpha_1\cdot\left(E-1\right)}{z_1}} \\ = \frac{1}{1 + tg^2\alpha_1 + \frac{2\pi^2}{3\cdot z_1^2}\cdot\left(\varepsilon_{12}-1\right)\cdot\left(2\cdot\varepsilon_{12}-1\right) \pm \frac{2\pi\cdot tg\alpha_1}{z_1}\cdot\left(\varepsilon_{12}-1\right)} \end{cases} \tag{4.9}$$

One starts in the relation (4.9) with the expression of the yield (4.6) for 4 pairs of engaging teeth, but immediately (even within the relation) we make a generalization by replacing the number 4 (four pairs of engaging teeth) with the variable E, which represents the full side of the coverage +1 and after the expressions written in the form of sums narrow down, the variable E is replaced with the degree of coverage, thus reaching the final shape. The average yield is more interesting than the instantaneous one and is calculated (precisely by integrating the instantaneous one from the minimum pressure to the maximum angle) simply by the approximation which determines the average yield by replacing in the expression of the instantaneous yield of the variable pressure angle (α_1) with its average value given by the normal pressure angle (standardized, α_0), (4.10), where ε_{12} represents the degree of coverage and is calculated with the expression (4.11) for the external engagement and the relation (4.12) for the inner engagement:

$$\eta_m = \frac{1}{1 + tg^2\alpha_0 + \frac{2\pi^2}{3\cdot z_1^2}\cdot\left(\varepsilon_{12}-1\right)\cdot\left(2\cdot\varepsilon_{12}-1\right) \pm \frac{2\pi\cdot tg\alpha_0}{z_1}\cdot\left(\varepsilon_{12}-1\right)} \tag{4.10}$$

$$\varepsilon_{12}^{a.e.} = \frac{\sqrt{z_1^2\cdot\sin^2\alpha_0 + 4\cdot z_1 + 4} + \sqrt{z_2^2\cdot\sin^2\alpha_0 + 4\cdot z_2 + 4} - \left(z_1 + z_2\right)\cdot\sin\alpha_0}{2\cdot\pi\cdot\cos\alpha_0} \tag{4.11}$$

$$\varepsilon_{12}^{a.i.} = \frac{\sqrt{z_e^2\cdot\sin^2\alpha_0 + 4\cdot z_e + 4} - \sqrt{z_i^2\cdot\sin^2\alpha_0 - 4\cdot z_i + 4} + \left(z_i - z_e\right)\cdot\sin\alpha_0}{2\cdot\pi\cdot\cos\alpha_0} \tag{4.12}$$

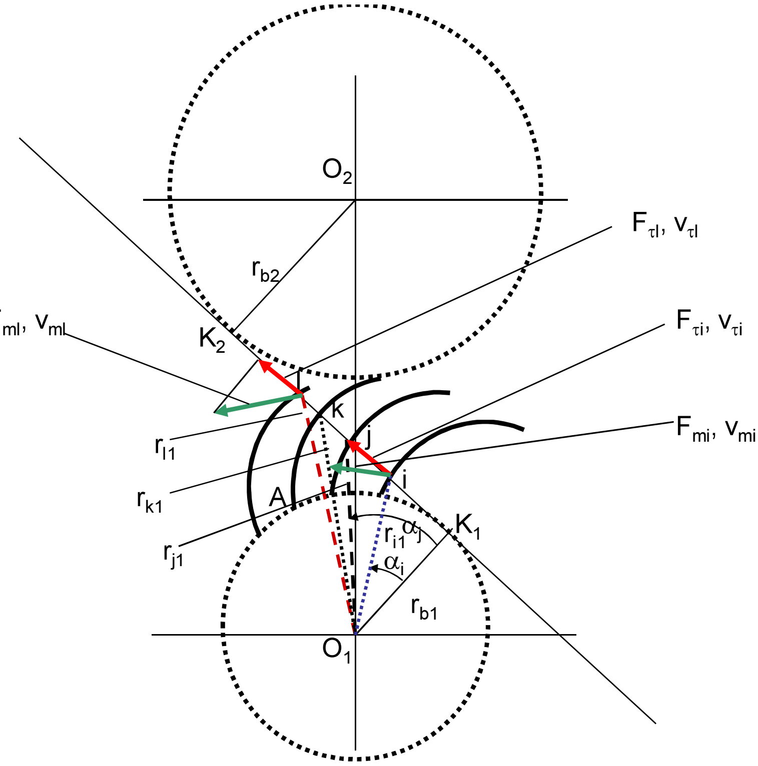

Fig. 11: Determining the yield of a gear with fixed axle gears (four pairs of simultaneous engagement teeth)

There are wheels of the helical gears, which are used very often (relations 4.13, 4.14, 4.15). For helical gears, the calculations show a decrease in the efficiency of the with the tilt angle on the rise of the teeth (β). For given angle which does not exceed 25°, the efficiency of fishing gear is good enough. However, when the tilt angle is greater than 25°, speeds will suffer a significant decrease in the yield:

$$\eta_m = \frac{z_1^2 \cdot \cos^2\beta}{z_1^2 \cdot (tg^2\alpha_0 + \cos^2\beta) + \frac{2}{3}\pi^2 \cdot \cos^4\beta \cdot (\varepsilon - 1) \cdot (2\varepsilon - 1) \pm 2\pi \cdot tg\alpha_0 \cdot z_1 \cdot \cos^2\beta \cdot (\varepsilon - 1)} \tag{4.13}$$

$$\begin{aligned} \varepsilon^{a.e.} = \frac{1 + tg^2\beta}{2 \cdot \pi} \cdot \Big\{ & \sqrt{\left[\left(z_1 + 2 \cdot \cos\beta\right) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3\beta \cdot \left(z_1 + \cos\beta\right)} \\ & + \sqrt{\left[\left(z_2 + 2 \cdot \cos\beta\right) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3\beta \cdot \left(z_2 + \cos\beta\right)} - \left(z_1 + z_2\right) \cdot tg\alpha_0 \Big\} \end{aligned} \tag{4.14}$$

$$\begin{aligned} \varepsilon^{a.i.} = \frac{1 + tg^2\beta}{2 \cdot \pi} \cdot \Big\{ & \sqrt{\left[\left(z_e + 2 \cdot \cos\beta\right) \cdot tg\alpha_0\right]^2 + 4 \cdot \cos^3\beta \cdot \left(z_e + \cos\beta\right)} \\ & - \sqrt{\left[\left(z_i - 2 \cdot \cos\beta\right) \cdot tg\alpha_0\right]^2 - 4 \cdot \cos^3\beta \cdot \left(z_i - \cos\beta\right)} - \left(z_e - z_i\right) \cdot tg\alpha_0 \Big\} \end{aligned} \tag{4.15}$$

Conclusion

There are wheels of the helical gears, which are used very often. For helical gears, the calculations show a decrease in the efficiency of the with the tilt angle on the rise of the teeth (β). For given angle which does not exceed 25°, the efficiency of fishing gear is good enough. However, when the tilt angle is greater than 25°, speeds will suffer a significant decrease in the yield.

The highest yield that can be achieved with two gears is that of the inner gear, with the inner driving gear (the wheel becomes the driver and the smaller wheel with external gear will be driven); Conversely, when we form an inner gear with the small (outer gear) driving wheel, the resulting yield is the smallest possible; When gearing is external, the output is higher for the high steering wheel; The more the normal engagement angle, α_0, decreases, the degree of coverage increases and with it the engagement efficiency; when the normal angle of engagement drops to 5 degrees, the coverage reaches 6.5-7.3 and the output reaches theoretical values of 99-99.5%, meaning that the gear will actually work at 100%. Yield increases also with the number of teeth of the driving wheel.

In order to avoid interference phenomenon, point A must lie between C and K_1 (the addendum circle of the wheel 2, C_{a2} need to cut the line of action between points C and K_1 and under no circumstances does not exceed the point K_1). Similarly, C_{a1} addendum circle must cut the action line between points C and K_2, resulting in point E, which in no circumstances, does not exceed the point K_2.

The conditions to avoid the phenomenon of interference can be written with the relations (3.1).

The basic conditions of interference, are the same (CA<K1C; CE<K2C), but the originality of this new presented method consist in the mode in which it was solved the classical relationship (see the system 3.1).

The system (3.3) represents a simple, unitary and general relationship capable of generating functional solutions for gears, giving the minimum number of teeth of wheel 1 (motor wheel) to avoid interference. In the appendix Table 1-15 of Figure 12 an alpha0 value (35°) will be chosen and the beta angles (from 0° to 40°) and the transmission ratio *i* (from 1 to 80) are incrementally incremented in order to thus getting the minimum number of teeth correctly.

Then, the alpha value (from 35° to 5°) will be decreased successively.

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Ethics

This article is original and contains unpublished material. Authors declare that are not ethical issues and no conflict of interest that may arise after the publication of this manuscript.

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Appendix, Figure 12 with 15 Tables

Figure 12, Table 1: α_0 = 35 [deg], β = 0 [deg]

α_0 [deg]	35		β [deg]	0					
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.8828	5.0325	5.1896	5.3204	5.4386	5.5467	5.6431	5.7198	5.7867	5.8439
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.8880	5.9243	5.9568	5.9805	5.9997	6.0158	6.0290	6.0389	6.0471	6.0539

Figure 12, Table 2: α_0 = 35 [deg], β = 10 [deg]

α_0 [deg]	35		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.7254	4.8679	5.0174	5.1419	5.2545	5.3576	5.4495	5.5226	5.5865	5.6411
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.6832	5.7178	5.7489	5.7715	5.7898	5.8052	5.8178	5.8273	5.8351	5.8416

Figure 12, Table 3: α_0 = 35 [deg], β = 20 [deg]

α_0 [deg]	35		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	4.2799	4.4022	4.5307	4.6379	4.7351	4.8240	4.9035	4.9667	5.0219	5.0693
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	5.1058	5.1358	5.1627	5.1824	5.1983	5.2116	5.2226	5.2308	5.2376	5.2432

Figure 12, Table 4: α_0 = 35 [deg], β = 30 [deg]

α_0 [deg]	35		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	3.6198	3.7136	3.8123	3.8949	3.9699	4.0388	4.1004	4.1495	4.1925	4.2294
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	4.2578	4.2812	4.3022	4.3176	4.3300	4.3404	4.3490	4.3554	4.3608	4.3651

Figure 12, Table 5: $\alpha_0 = 35$ [deg], $\beta = 40$ [deg]

α_0 [deg]	35		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	2.8475	2.9104	2.9769	3.0327	3.0836	3.1304	3.1724	3.2060	3.2355	3.2608
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	3.2804	3.2965	3.3110	3.3216	3.3302	3.3374	3.3433	3.3477	3.3514	3.3545

Figure 12, Table 6: $\alpha_0 = 20$ [deg], $\beta = 0$ [deg]

α_0 [deg]	20		β [deg]	0						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	12.323	12.966	13.624	14.161	14.637	15.066	15.443	15.740	15.997	16.215
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	16.382	16.519	16.641	16.730	16.802	16.862	16.911	16.948	16.978	17.003

Figure 12, Table 7: $\alpha_0 = 20$ [deg], $\beta = 10$ [deg]

α_0 [deg]	20		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	11.835	12.447	13.075	13.586	14.041	14.450	14.810	15.094	15.339	15.547
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	15.706	15.837	15.954	16.039	16.107	16.164	16.211	16.246	16.275	16.299

Figure 12, Table 8: $\alpha_0 = 20$ [deg], $\beta = 20$ [deg]

α_0 [deg]	20		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	10.446	10.994	11.535	11.977	12.370	12.725	13.036	13.282	13.495	13.676
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	13.814	13.927	14.028	14.102	14.161	14.211	14.252	14.282	14.308	14.328

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Figure 12, Table 9: $\alpha_0 = 20$ [deg], $\beta = 30$ [deg]

α_0 [deg]	20		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	8.4777	8.8841	9.3024	9.6448	9.9497	10.225	10.468	10.659	10.825	10.966
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	11.074	11.163	11.242	11.299	11.346	11.385	11.417	11.441	11.461	11.477

Figure 12, Table 10: $\alpha_0 = 20$ [deg], $\beta = 40$ [deg]

α_0 [deg]	20		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	6.2280	6.5020	6.7854	7.0182	7.2263	7.4147	7.5813	7.7128	7.8269	7.9241
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	7.9985	8.0597	8.1143	8.1540	8.1862	8.2131	8.2352	8.2517	8.2654	8.2767

Figure 12, Table 11: $\alpha_0 = 5$ [deg], $\beta = 0$ [deg]

α_0 [deg]	5		β [deg]	0						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	176.52	188.86	201.22	211.13	219.81	227.54	234.28	239.55	244.09	247.93
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	250.85	253.24	255.37	256.92	258.17	259.21	260.06	260.70	261.23	261.67

Figure 12, Table 12: $\alpha_0 = 5$ [deg], $\beta = 10$ [deg]

α_0 [deg]	5		β [deg]	10						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	168.66	180.45	192.25	201.72	210.01	217.38	223.83	228.86	233.19	236.86
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	239.65	241.94	243.97	245.45	246.64	247.63	248.45	249.06	249.57	249.98

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Figure 12, Table 13: α_0 = 5 [deg], β = 20 [deg]

α_0 [deg]	5		β [deg]	20						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	146.73	156.95	167.20	175.42	182.62	189.03	194.63	198.99	202.76	205.94
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	208.37	210.35	212.12	213.40	214.44	215.30	216.01	216.54	216.98	217.34

Figure 12, Table 14: α_0 = 5 [deg], β = 30 [deg]

α_0 [deg]	5		β [deg]	30						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	115.16	123.15	131.16	137.58	143.22	148.23	152.61	156.03	158.97	161.47
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	163.37	164.92	166.30	167.31	168.12	168.79	169.35	169.76	170.11	170.39

Figure 12, Table 15: α_0 = 5 [deg], β = 40 [deg]

α_0 [deg]	5		β [deg]	40						
i	1	1.25	1.6	2	2.5	3.15	4	5	6.3	8
z_{min}	80.086	85.602	91.136	95.575	99.465	102.93	105.96	108.33	110.36	112.09
i	10	12.5	16	20	25	31.5	40	50	63	80
z_{min}	113.40	114.48	115.43	116.13	116.69	117.16	117.54	117.83	118.07	118.26