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The Application of Numerical Taxonomy Technique in the Iron Ore Flotation to Determine Appropriate pH and Particle Size Distribution

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Abstract: The numerical taxonomic technique is one of the multivariate decision-making techniques for assessment and ranking. It is widely used for planning and development studies. In this research, flotation experiments were conducted for Band Narges mine iron ore using different pH and particle size ranges. Numerical taxonomy technique, as one of the most important multi-attribute decision-making techniques, was used to determine the best range of particle sizes for the input feed of the flotation cell as well as the pH of the pulp. For this purpose, two different particle size ranges were selected for six different pH values as options. Criteria for evaluating particle size ranges and pHs were determined. Subsequently, effective criteria were determined by performing various tests. Finally, the ranking of the tests was determined using numerical taxonomy. Based on the F_i value (a parameter indicating the appropriate status or developmental rate of the option), the particle sizes <74 μm and pH = 9 were proposed as the optimum conditions to float Band Narges Iron Ore.

Keywords: Numerical Taxonomy, Flotation, pH, Particle Size Distribution, Iron Ore, Classification

Introduction

Froth flotation is a complex physicochemical separation process based on the difference in the surface properties of minerals that is commonly used to separate valuable minerals and unwanted gangue minerals. Improving knowledge and understanding of the process leads to significant progress in achieving higher grade and recovery. However, the amount of energy used in the flotation process is not as much as the grinding process, but its optimization would reduce energy consumption. In recent years, the advent of gigantic float cells and energy-saving competition to reduce operating costs have led to efforts to optimize energy consumption. Energy is used to rotate agitators in mechanical flotation cells. The rotation of the agitator in the flotation cell gives the energy required to suspend the particles, distribute the bubbles and produces an interaction between the bubbles and the particles. Part of the energy consumes to rotate the pulp inside the cell, but a large part of the energy is wasted by micro-interactions between the three-phase include air, water and solids (Ahmed and Jameson, 1985; Amini *et al*., 2013; Chau, 2009; Deglon *et al*., 1998; Drzymala and Kowalczuk, 2018; Newell and Grano, 2006; Pyke *et al*., 2003; Reis *et al*., 2019; Schubert and Bischofberger, 1978; 1998).

Flotation performance depends on a variety of parameters, including distribution of particle size, solid percent pulp, reagent dosage, *Eh* and pH. In practice, selectivity in complex separation depends on the balance between reagent dosage and pH (Bahrami *et al*., 2019; Dehghani *et al*., 2013; Rezai *et al*., 2010; Tan *et al*., 2018; Zanin *et al*., 2019). The floatability of the sulfide minerals is strongly related to the potential difference of the mineral-solution interface (pulp potential). It is possible to float or dissipate minerals by oxidation or reduction. The measured potentials, therefore, determine whether or not the mineral will float. It is between the potential of the minerals and the potential of the solution. In the same solution, different materials have different *Eh* values. Controlling *Eh*-pH data in an operating process can help to reduce reagent additions and provide useful additional and useful information to solve the operational problem of flotation plants. Some studies related to *Eh*/pH have shown that *Eh* is an appropriate indicator of mineral floatability (Chimonyo *et al*., 2017; Göktepe, 2002; Liu *et al*., 1994).

Particle size is another important factor that plays a critical role in the flotation performance. It affects the probability of the particles colliding with the bubbles, the attachment of the particles to the bubbles after the collision and the stability of the attachment (Eskanlou *et al*., 2018; Ross, 1997). The rate of recovery of fine particles is low due to a lower probability of particle-bubble collisions, prone to entrainment and large specific areas, which may lead to excessive reagent adsorption and other effects associated with chemically active particles. On the other hand, there is an essential difficulty in the flotation of coarse particles in the high bubble-particle detachment efficiency (Pyke *et al*., 2003), as well as a decrease in the buoyancy of the particle-bubble aggregate relative to the pulp. Furthermore, increasing particle sizes may result in longer induction times and a commensurate deterioration in floatability (Eskanlou *et al*., 2019; Jameson *et al*., 1977).

Previous studies showed that some multi-attribute decision-making methods, such as AHP and TOPSIS, have been used to select appropriate methods and parameters in the mining, mineral processing industries. In one study, AHP and PROMETHEE methods were used to determine the type and concentration of coagulants for the physical and chemical purification of wastewater in a textile factory (Aragonés-Beltrán *et al*., 2009). Another study used the Fuzzy TOPSIS method to select the type of collector and optimize the dosage of it in the flotation of lead and zinc sulfide minerals (Kostović and Gligoric, 2015). Some researches utilized Fuzzy TOPSIS (Safari *et al*., 2012) and the AHP methods (Safari *et al*., 2010) to determine the suitable location for the construction of a processing plant for an iron ore mine. Nowever this method was used wildy in other fields to make a final decision, but the number of research that used the method in mineral processing, especially in flotation process is limited. Thus, in this research, it was applied to determine the appropriate *ph* and particle sizes.

Selecting the appropriate pH and particle size range can improve flotation performance by increasing grade and recovery. The applicable particle size ranges and pH pulp for the Band Narges iron ore flotation process were therefore investigated and determined using a mathematical method based on Multi-Attribute Decision-Making techniques (MADM). For this purpose, pH at six different levels and particle size distribution at two different levels have been considered. Effective parameters have been determined. Finally, the optimum distribution of particle sizes and PH values was proposed using the mathematical method of numerical taxonomy.

Research Methodology

Representative samples for the experiments were taken from the Band Narges iron ore mine located 60 km northeast of Badrud, Isfahan City, Iran. The degree of freedom of iron ore (d_{80}) was determined as 150 µm by mineralogical studies. In the next step, the samples were ground and floated without pre-concentration. Samples were ground to a size of less than 150 µm. Then, six flotation series tests were carried out at different pHs for a particle size distribution of 0-150 µm. Each test series was repeated three times and the average amount was calculated.

Taxonomy Analysis Method

One of the multi-attribute decision-making methods is a taxonomic analysis method. It was first proposed by Adenson in 1763 and was extended to a group of mathematicians in 1950. Taxonomy analysis has been used in different sciences for different categories and its specific type is numerical taxonomy. Numerical taxonomy is used to assess the similarity and proximity between taxonomic units to classify these elements into taxonomic groups. This method is based on an analysis of existing alternatives using several criteria and provides a complete ranking for the evaluation of options. The method is widely used in various engineering fields (Feoli and Ganis, 1984; Soltanpanah *et al*., 2010; Rahman *et al*., 2013; Yari *et al*., 2015).

One of the advantages of using the numerical taxonomy method in comparison with other multi-attribute decision-making methods is that it is not necessary to determine the relative importance of criteria based on expert judgments in this method. As a result, expert judgments have less interference with the analysis of the data so the results are more accurate than others. Taxonomy analysis includes eight steps as follows (Sneath and Freeman, 1975).

Step 1: Identify Options and Set Assessment Criteria

At first step, experts shall identify and determine the m alternative (A_2, A_1, ..., A_m) and the n assessment criteria (C_1, C_2, ..., C_n) in accordance with the problem under consideration.

Step 2: Set up the Decision Matrix and Calculate the Mean and Standard Deviation of the Data

Subsequently, after evaluating the alternatives based on different criteria, the decision matrix is established, as shown in Table 1. In this table, r_{ij} represents the amount of the ith option in quantitative or qualitative terms based on the jth criterion. After the data matrix is formed, the mean and standard deviation of each column are calculated.

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Table 1: Decision matrix

Options criterion	C_1	C_2	$C3$	.	.	C_n
A_1	r_{11}	r_{12}	r_{13}	.	.	r_{1n}
A_2	r_{21}	r_{22}	r_{23}	.	.	r_{2n}
.	.	.	.	.	.	
.	.	.	.	.	.	
A_n	r_{m1}	r_{m2}	r_{m3}	.	.	r_{mn}
Mean	X_1X_2			.	.	X_n
Standard Deviation	σ_1	σ_1	.	.	.	σ_n

Step 3: Formation of Standard Matrix (Normalized)

In the decision matrix, the options are expressed in terms of criteria that have different measurement scales. At this stage, attempts are being made to eliminate their various units (measuring scales) or to make them dimensionless. The equation below is used to get this purpose:

$$Z_{ij} = \frac{x_{ij} - \bar{x}_j}{\sigma_j} \tag{1}$$

$\bar{x}_j$: The mean of each matrix column

σ_j : The standard deviation of each matrix column

It should be noted at this stage that criteria with a negative aspect (cost criteria) must be reversed or, in other ways, considered negative. Finally, in conventional methods, for each column of the standard matrix, the maximum positive or negative criteria will be chosen as ideal positive or negative. It's defined as the DO_j.

Step 4: Determine the Compound Distance Between Options

At this stage, based on the standard matrix, the distance of each option is determined from the following equation:

$$D_{ab} = \sqrt{\sum_{j=1}^{m}} \left(Z_{aj} - Z_{bj}\right)2 \tag{2}$$

where, *a* and *b* are two options to be assessed. This operation is a kind of paired comparison of the two options. The main features of the operation are as follows:

1) The distance of both options is zero ($D_{ab} = D_{ba} = 0$)
2) The distance between *a* and *b* is equal to the distance *b* of *a*. ($D_{ab} = D_{ba}$)

According to the above items, the compound distance matrix is formed for options that the main diameter represents the distance of each option with itself is equal to zero.

Step 5: Determine the Shortest Distance

The minimum distance in each row of the matrix is determined in this step. Then, their mean and standard deviations are obtained. For the shortest distance, the same method is used.

Step 6: Homogenizing Options

There may be options that have much more or far fewer distances than other options. Therefore, heterogeneous options should be removed them from the set. To this end, the upper and lower limits are obtained using the following equations:

$$O_r = \bar{d}_r \pm 2\sigma_{dr} \tag{3}$$

$$O_r(+) = \bar{d}_r \pm 2\sigma_{dr} \quad \textit{upper limit} \tag{4}$$

$$O_r(-) = \bar{d}_r \pm 2\sigma_{dr} \quad \textit{lower limit} \tag{5}$$

In this case, the d_r has been coordinated between the upper and lower limits. The options outside this specified range must be deleted. The decision matrix is again established without the deleted options. The steps are repeated.

Step 7: Determination of Options Patterns

At this stage, the distance between each option is calculated from the ideal value (as specified in step 4). The short distance from the ideal indicates the appropriate condition of the option and a large distance indicates its incorrect status. Option patterns are also derived from the following equation:

$$C_{io} = \sqrt{\sum_{j=1}^{m}} \left(Z_{ij} - Z_{bj}\right)2 \tag{6}$$

Step 8: Ranking the Developmental Rate of Options (F_i)

The ranking of the development rate and the status of the options are discussed at this stage. If F_i is the developmental rate of an option (the appropriate status of an option), then:

$$F_i = \frac{Cio}{Co} \tag{7}$$

where, C_{io} is a pattern of each option and Co is the upper limit of the development. To calculate Co, the mean and deviation of Co should be determined. This was done at the end of step seven and calculated as follows:

$$Co = \overline{Cio} + 2\sigma_{cio} \tag{8}$$

The F_i is between zero and one. If it is closer to zero, it indicates more development and better conditions for the option. If it is closer to one, it indicates its bad status and the lack of potential development of the option. In this case, the taxonomic technique is completed and the options ranking is specified.

Discussion

Flotation is one of the most important mineral arrangement methods established for particles in the fluid and airflow environments. Airflow causes bubbles in the pulp. Increasing the free surface of the particles (at constant mass), causes more contact and stronger bonding to the air bubble and vice versa. By increasing the mass of the particles when the free surface is constant, the force of gravity increases and the probability of separation increases (Rahimi *et al*., 2012; Van Deventer *et al.*, 2004). A number of flotation tests were carried out on particle sizes of less than 150 µm with 500 g/ton Sodium Silicate as a dispersant, Dirol and Alke 724 with values of 200 and 600 g/ton as frother and collector, respectively, to investigate the appropriate flotation size ranges. Lime was used to adjust the pH. The results of the flotation are shown in Table 2. As it can be seen from the table at pH equal to 9 showed the best recovery and grade, 86.94 and 48.24%, respectively.

The next set of flotation experiments was performed for particle sizes below 74 µm with different pH values. For this purpose, ore samples were ground to 0 to 74 µm in size. The results are shown in Table 3.

The optimum particle size range, as well as pH, were determined using a mathematical method based on MADM called the numerical taxonomy mathematical method, after measurement and calculation of concentrate and tailing grades and recovery for all tests.

As can be seen from Table 2 and Fig. 1, at pH = 9 for particle size fraction 0-150 µm, we have the highest recovery and grade levels (86.94 and 48.24%, respectively). As pH increases, the grade and recovery of flotation decrease. Table 3 and Fig. 2 also show a similar trend for particle size fractions between 0-74 µm as the highest recovery and grade are at pH = 9, 86.74 and 49.52%, respectively. According to Table 2 and 3, 0-74 µm fraction has higher floatability; however, 0-150 µm fraction shows higher recovery.

We deal with a large number of data during the flotation process, so it is difficult to choose the data that can show optimum conditions. We, therefore, analyze and rank the data using a mathematical multi-attribute taxonomy decision-making method to overcome the difficulties. In the next step, the data with the highest-ranking using this method will be selected as an appropriate condition for the flotation process.

Determination of Appropriate Particles Sizes and pH using the Numerical Taxonomic Multi-Attribute Decision-Making Technique

In this study, the proper particle size of the input feed of the flotation cells and the pH of the pulp will be determined based on the following model Fig. 3.

Step 1: Select Options

Initially, to study the effect of particle size ranges and pH, flotation tests were conducted for two particles sizes fractions (0-74 and 0-150 µm) and 6 different pH values (9, 9.5, 10, 10.5, 11, 11.5). They were used as alternative options according to the first step, Fig. 3.

Step 2: Determination of Appropriate Criteria to Evaluate Particles Sizes Fractions and pH

Criteria for assessing the particle size ranges and pH values have been determined. They were shown with the related symbols and their effects in Table 4. According to Table 4, both C_1 and C_3 have positive effects. The higher the criteria for an option, the higher the priority of that option. On the other hand, C_2 has a negative impact as the number of criteria for one option increases, the priority level of the choice option decreases. Also, all criteria for small quantities have been determined based on the tests.

Step 3-8: Formation of Decision Matrix and Use of Numerical Taxonomy

At this stage, the measured values of the C_1, C_2, C_3 for different pH values (1 to 12) are considered to be the decision matrix. In this case, a decision matrix with a dimension of 3*12 was obtained. Where 12 represents the number of options (alternative) and 3 represents the number of criteria that were presented in Table 5. The numerical taxonomic method was used to evaluate the parameters obtained from different experiments in order to select the best particle size ranges for the input feed of the flotation cell and the best pH value for the pulp. For the next steps, the taxonomic procedure for the decision matrix were performed. The results were shown in Tables 6-10.

The mean and standard deviation of the columns for each decision matrix were calculated Table 6. To normalize the matrix, Eq. 1 was used. As a result, standard matrixes have been obtained, see Table 7. In

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the next step, the largest positive number of each column for positive aspect criteria and the largest negative number of each column for negative aspect criteria were selected. They were identified as ideal (DOj). Equations 2 and 3 were used to determine the compound distance between the alternatives for each of the criteria. The results are shown in Table 8.

Table 2: The effect of pH on the flotation of iron ore particles with size range 0-150 µm

Recovery%	Tailing grade%	Concentrate grade%	pH
86.94	26.02	48.24	9.0
80.81	34.11	46.40	9.5
70.31	36.09	47.46	10.0
66	38.10	46.75	10.5
41.37	41.45	46.50	11.0
46.18	41.42	45.96	11.5

Table 3: The effect of pH on the flotation of iron ore particles with size range 0-74 µm

Recovery%	Tailing grade%	Concentrate grade%	pH
86.74	24.00	49.52	9.0
80.26	31.06	48.10	9.5
71.93	34.06	48.60	10.0
76.9	33.22	47.80	10.5
51.73	39.80	47.40	11.0
57.91	57.91	46.20	11.5

Table 4: Evaluation criteria for particles size ranges and pH

Describing criteria	Aspect effect	Symbol	Criteria title
The higher the concentrate's%, the higher the quality of the product	Positive	C_1	Concentrate grade %
The higher waste's%, the higher percentage of valuable mineral is lost	Positive	C_2	Waste grade %
The higher recovery, the lower waste of valuable mineral	Negative	C_3	Recovery %

Table 5: Effect of different particle size ranges and different PHs on flotation performance (decision Matrix)

Particle sizes (µm)	pH	Indicator C_1	C_2	C_3
0-74	pH1	86.74	24.00	49.52
	pH2	80.26	31.06	48.10
	pH3	71.93	34.06	48.60
	pH4	76.90	33.22	47.80
	pH5	51.73	39.80	47.40
	pH6	57.91	57.91	46.20
0-150	pH7	86.94	26.02	48.24
	pH8	80.81	34.11	46.40
	pH9	70.31	36.09	47.46
	pH10	66.00	38.10	46.75
	pH11	41.37	41.45	46.50
	pH12	46.18	41.42	45.96

Table 6: The mean and standard deviation of each column

	C_1	C_2	C_3
Mean value of pH	47.41	34.95	68.09
Standard deviation of pH	1.042	5.51	19.94

Table 7: Standard matrix and determination of ideal positive and negative numbers

pH	C_1	C_2	C_3
pH1	2.024	-1.987	1.248
pH2	0.662	-0.706	0.815
pH3	1.142	-0.162	0.257
pH4	0.347	-0.314	0.590
pH5	-0.010	0.880	-1.095
pH6	-1.161	0.927	-0.681
pH7	0.796	-1.621	1.262
pH8	-0.969	-0.152	0.851
pH9	0.048	0.207	0.149
pH10	-0.633	0.572	-0.140
pH11	-0.873	1.180	-1.788
pH12	-1.391	1.174	-1.467
Doi	2.024	-1.987	1.262

Table 8: The compound distance between alternatives and determining the minimum distance

	pH1	pH2	pH3	pH4	pH5	pH6	pH7	pH8	pH9	pH10	pH11	pH12	Minimum distance	Mean of minimum	Standard deviation
pH1	0.000	1.920	2.257	2.44	4.225	4.729	1.281	3.533	3.151	3.947	5.258	5.388	1.281	0.86	0.263
pH2	1.920	0.000	0.915	0.536	2.572	2.868	1.027	1.722	1.286	2.055	3.562	3.600	0.536		
pH3	2.257	0.915	0.000	0.860	2.060	2.714	1.805	2.193	1.160	1.961	3.169	3.342	0.850		
pH4	2.440	0.536	0.850	0.000	2.100	2.348	1.529	1.378	0.756	1.527	3.073	3.092	0.536		
pH5	4.225	2.572	2.060	2.100	0.000	1.224	3.530	2.403	1.470	1.182	1.147	1.460	1.182		
pH6	4.729	2.868	2.714	2.348	1.224	0.000	3.755	1.885	1.634	0.835	1.152	0.854	0.835		
pH7	1.281	1.027	1.805	1.529	3.530	3.750	0.000	3.755	2.267	2.969	4.465	4.476	4.476		
pH8	3.533	1.722	2.193	1.387	2.403	1.885	2.332	0.000	1.287	1.273	2.959	2.704	1.273		
pH9	3.151	1.286	1.160	0.756	1.415	1.634	2.267	1,287.000	0.000	0.825	2.355	2.370	0.756		
pH10	3.942	2.055	1.961	1.527	1.182	0.835	2.969	1.273	0.825	0.000	1.773	1.642	0.825		
pH11	5.258	3.562	3.169	3.073	1.147	1.152	4.465	2.959	2.355	1.773	0.000	0.610	0.610		
pH12	5.388	3.6	3.342	3.092	1.460	0.854	4.476	2.704	2.370	1.642	0.610	0.000	0.610		

Table 9: Determine the distance of options from ideal values

pH	Distance from DO1	Distance from DO2	Distance from DO3	Total squared difference	Cio
pH1	0.000	0.000	0.000	0.000	0.000
pH2	1.855	1.641	0.200	3.696	1.922
pH3	0.779	3.332	1.010	5.121	2.263
pH4	2.722	2.799	0.452	5.973	2.444
pH5	4.136	8.221	5.556	17.912	4.232
pH6	10.143	8.494	3.777	22.413	4.734
pH7	1.507	0.134	0.000	1.642	1.281
pH8	8.958	3.366	0.169	12.492	3.534
pH9	3.905	4.813	1.240	9.958	3.156
pH10	7.060	6.547	1.965	15.573	3.946
pH11	8.393	10.028	9.305	27.726	5.266
pH12	11.662	9.993	7.445	29.101	5.395

Table 10: Development rate (F_i) and rating options for the flotation process

pH	F_i	Rank
pH1	0.000	1
pH2	0.303	3
pH3	0.356	4
pH4	0.358	5
pH5	0.666	9
pH6	0.745	10
pH7	0.202	2
pH8	0.556	7
pH9	0.479	6
pH10	0.621	8
pH11	0.829	11
pH12	0.829	12

Equations 5 and 6 were used to homogenize the options and the upper and lower limits for each of the matrixes were obtained. To determine the pattern of options (*Cio*) in each matrix, at this stage, the distance of each option is calculated from the ideal values (*DOj*) as shown in Table 9. As noted, the low distance from the ideal value is an appropriate condition for this option. In the last step, the upper limit of development (*Co*) was calculated for data based on Eq. 9. To rank the development rate of the options, an index (F_i) was obtained for each of them by using Eq. 8. This index is between zero and one. Whenever the indicator is closer to zero, it indicates the better development of the option (better situation) and higher ranking. Whenever the indicator is closer to one, it indicates its poor status and lower ranking. The results of these calculations have been shown in Table 10. According to Table 10, the lowest rating for flotation is in the case of particle size of 0-74 µm and a pulp pH of 9, which is the first rank. This option is recommended as an appropriate condition for flotation using numerical taxonomy.

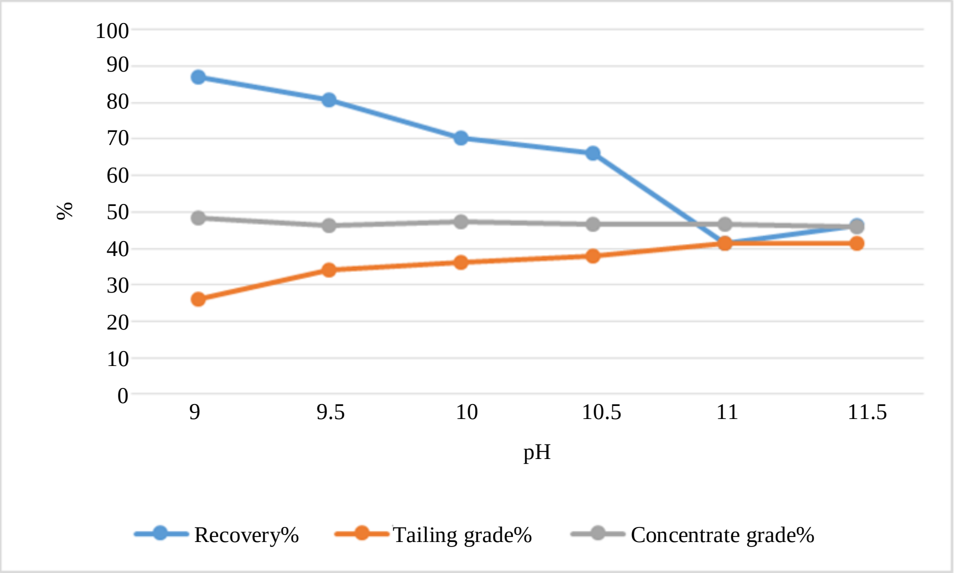

Fig. 1: pH changes and its effect on flotation of iron ore particles with size range 0-150 µm

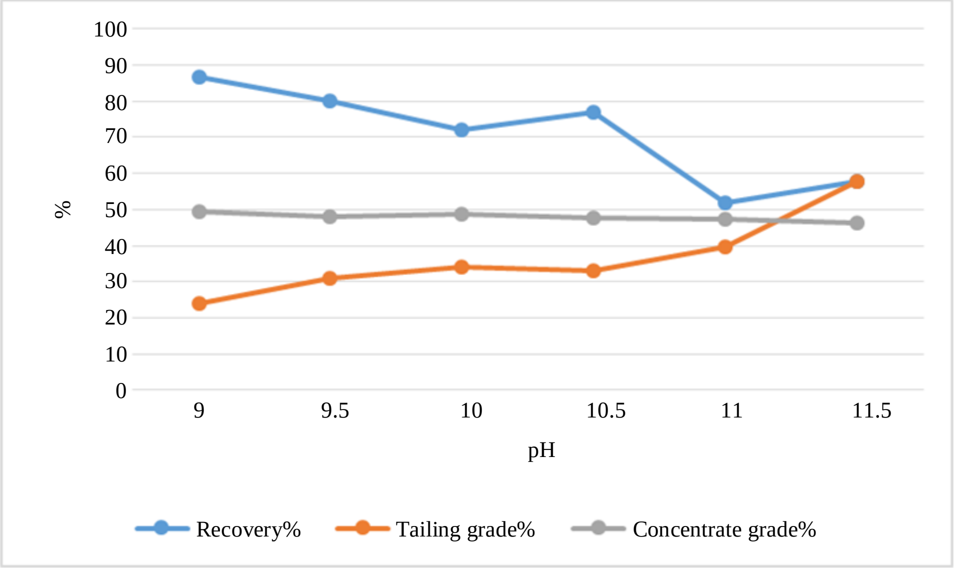

Fig. 2: pH changes and its effect on grade and recovery (particle sizes 0-74 µm)

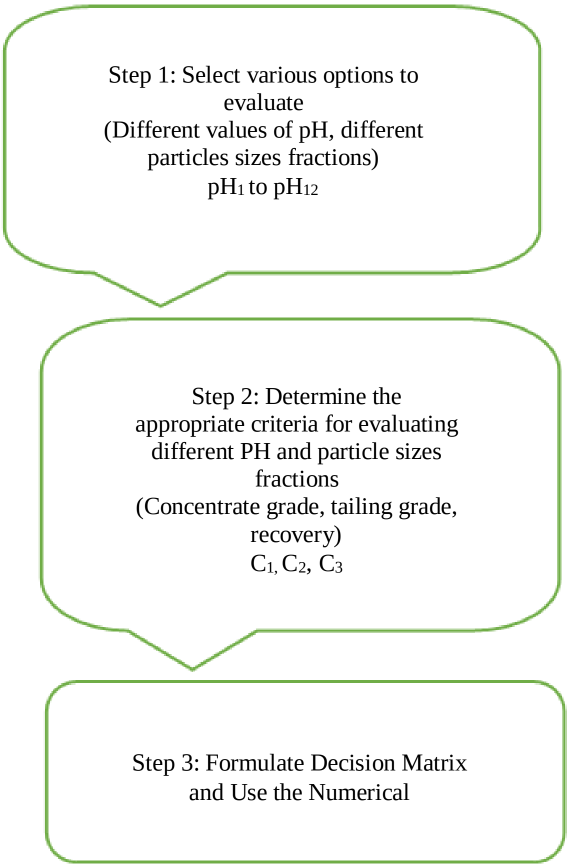

Fig. 3: The steps to select appropriate particles sizes fraction and pH of pulp in the flotation process

Conclusion

Consideration and evaluation of key factors, such as pH and particle size ranges, will enable us to have adequate knowledge of these parameters to optimize the flotation process. It is, therefore, necessary to classify and use the appropriate technique in order to understand the relationship between the parameters and finally their effects on flotation performance. Numerical taxonomy is one of the techniques most widely used to discover this relationship. Numerical taxonomy analysis is an excellent method of grading, classifying and comparing different parameters in engineering science, especially in mineral processing.

In this study, it was noted that factors such as pH and particle size ranges have a significant impact on the flotation process. The smaller the particles, the greater the degree of freedom of valuable minerals. An increased degree of particle freedom will, however, increase the flotation performance, but decreasing the particle size from certain ranges will have a negative effect on the flotation performance. Determining appropriate particle sizes with an appropriate degree of freedom is, therefore, an important issue for mineral processing plants. In addition, the surface of the particles should be properly charged in order to have an appropriate flotation performance. So pH is another important parameter.

In this research, numerical taxonomic mathematical methods clearly showed the effect of pH and particle size in the flotation process. This method has been shown experiments with similar pH value and smaller particles have a higher ranking. In other words, they have higher flotation performance. The results showed the particle sizes <74 µm and pH = 9 were proposed as the optimum conditions to float Band Narges Iron Ore based on the Fi value. In addition, according to the results, using a mathematical method such as numerical taxonomy is a strong tool to evaluate flotation performance.

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Authors Contributions

Rasoul Khosravi, Fahimeh Dehghani and Hossein Siavoshi: Participated in all experiments, coordinated the data-analysis. Coordinated the data-analysis and contributed to the writing of the manuscript. Coordinated the data-analysis and contributed to the writing of the manuscript.

Amir Pazoki, Reza Jahanian and Tathagata Ghosh: Coordinated the data-analysis. Coordinated the data-analysis. The writing of the manuscript.

Ethics

This article is original and contains unpublished material. On behalf of all authors, the corresponding author states that there is no conflict of interest.

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