Why College or University Students Hate Proofs in Mathematics?

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Abstract: Problem Statement: A proof is a notoriously difficult mathematical concept for students. Empirical studies have shown that students emerge from proof-oriented courses such as high-school geometry, introduction to proof, complex and abstract algebra unable to construct anything beyond very trivial proofs. Furthermore, most university students do not know what constitutes a proof and cannot determine whether a purported proof is valid. A proof is a convincing method that demonstrates with generally accepted theorem that some mathematical statement is true and each proofs step must follow from previous proof steps and definition that have already been proved. To motivate students hating proofs and to help mathematics teachers, how a proof can be taught, we investigated in this study the idea of mathematical proofs. **Approach:** To tackle this issue, the modified Moore method and the researcher method called Z.Mbaïtiga method are introduced follow by two cases studies on proof of triple integral. Next a survey is conducted on fourth year college students on which of the proposed two cases study they understand easily or they like. **Results:** The result of the survey showed that more than 95% of the responded students pointed out the proof that is done using details explanation of every theorem used in the proof construction, the case study2. **Conclusion:** From the result of this survey, we had learned that mathematics teachers have to be very careful about the selection of proofs to include when introducing topics and filtering out some details which can obscure important ideas and discourage students.

Keywords: Why proofs, mathematics proofs. moore and Z. Mbaïtiga methods

INTRODUCTION

When making a comparison between mathematics and other subject, we can say with certainty that in mathematics things are proved; while in other subjects they are not. This statement needs certain qualifications, but it does express the difference between mathematics and other sciences. In most fields of study knowledge is acquired from observations, by reasoning about the results of observations and by studying the observations, methods and theories of others. Mathematics was once like this too. Ancient Egyptian, Babylonian and Chinese mathematics consisted of rules for measuring land, computing taxes, predicting eclipses, solving equations. Methods were learnt from the observations and handed down to others. Modern school mathematics is still often practiced in this way.
But there were changes in the approach to mathematics. The ancient Greeks have found that in arithmetic and geometry it is possible to prove that results were true. They have found that some truths in mathematics were obvious and that many of the others could be shown to follow logically from obvious ones. Pythagoras' theorem Eq.1 on right-angle triangle shown in Fig.1 for

example is not obvious, but a way was found of deducing it from geometrical facts that were apparent. For example: now let a and b of Fig.1a be 5 and 12 in Fig.1b, find the value of *c* then prove that Eq. 1 is true.

Theorem: If a triangle has sides of length (a, b, c), with enclosing an angle of 90 ° (right angle), then:

$$a^2 + b^2 = c^2 \qquad (1)$$

Finding the value of c: Organization of information:

$$\begin{cases} a = 5; a^2 = a \times a = 25 \\ b = 12; b^2 = b \times b = 144 \\ a^2 + b^2 = 25 + 144 = 169 \end{cases} \qquad (2)$$
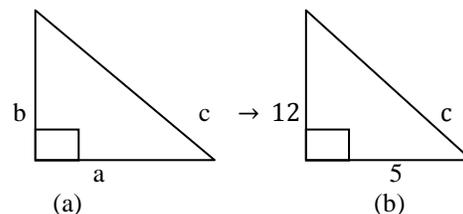

Fig.1: Right triangle with legs a and b

Replacing the value of $a^2 + b^2$ of Eq.2 into Eq.1 and deduces the value of c:

$$\begin{cases} a^2 + b^2 = c^2 \\ \underbrace{1 \quad 6 \quad 9} = c^2 \\ \sqrt{169} = c \xrightarrow{\text{yields}} 13 \end{cases} \qquad (3)$$

- Proof of Eq. 1:

$$\begin{cases} a^2 + b^2 = c^2 \\ \underbrace{1 \quad 6 \quad 9} = c^2 \\ 169 = 169 \end{cases} \qquad (4)$$

- Conclusion: Eq.1 is true.

But why a is equal to 5 and b equal to 12? Instead of 3 and 6? If a is equal to 3 and b equal to 6 really Eq.1 can be proved? The idea behind these questions is that, mathematics is not about answers, it is about processes to understand why a result is true, hence the importance of proof.

At first it was hoped that every subject would become like mathematics, with all the truths following obvious true basic statements. This did not happen, Physics, Biology, Economics and other Sciences discover general truths, but to do so they rely on observations. The theory of relativity is not proved true; it is tested against observations. As a result, mathematic has always been regarded as having a different kind of certainty that obtainable in other sciences. If a scientific theory is accepted because observations have agreed with it, there is always in principle a small doubt that a new observation will not agree with the theory, even if all previous observations have agreed with that theory. If a result is proved correctly, that cannot happen.

For more than two thousand years mathematics has attracted those who valued certainty and has served as the supreme example of certain knowledge. It has also attracted those who wanted knowledge that did not rely on the authority of others; a moment's thought will reveal how little of our knowledge is like this. Can we be sure, however that the steps in our reasoning are - correct? Are we really sure that what seems obvious to us is true? Can we expect all mathematical truths to follow from the obvious ones? Does really mathematics proofs stimulate the asleep cerebral nerves? If someone can prove one of the above questions mathematically

then his or her contribution to the mathematic world will be appreciated. These questions are not easily answered and must be left until after some examples of proofs have been proved. For students, what is really difficult in mathematic proof is the concept of proof. The difficulty manifests itself in three principal points:

- Appreciating why proofs are important
- The tension between verification and understanding
- Proof construction

The first point describes a spurious but convincing proof and a correct but unconvincing proof of deep result in linear algebra. The second point illustrates an underlying proof template that assist in the development of proof technique in much the same way as a sense of perspective is essential for the ability to draw well. The third point describes the manipulation of the theorems or definitions involve in the proof construction.

MATERIALS AND METHODS

What does proof mean and its role in mathematics?: To this question many mathematics teachers would consider the answers straightforward: A mathematical proof is a formal and logical line of reasoning that begins with a set of axioms and moves through logical steps to conclusion. And the purpose of proving a theorem is to establish its mathematical certainty. A proof confirms truth for a mathematician the way experiment or observations does for the natural scientist [1]. Such views are commonly held by mathematics teachers and are passed along to students. However, many mathematics professors and some mathematicians believe that proofs are much more that this. Davis and Hersh [2] argue that it is probably impossible to define precisely what types of argument will be accepted as a valid proof by the mathematical community. There are some aspects of proof that distinguish it from other types of arguments. As an example, proofs about a concept must use the concept's definition and must proceed deductively, as opposed to examining prototypical a cases or giving an intuitive arguments. And if a result is incorporated in a proof that result must accepted by mathematical community [3].

Beyond this, some mathematics teachers or educators argue that whether or not an argument is accepted as a proof depends not only on its logical structure but also on how convincing the argument is. At different places in the mathematics educators, a

proof has been defined as an argument that convinces an enemy [4], an argument that convince mathematician that knows well the subjects, or an argument that suffices to convince a reasonable septic [5]. Other who focuses on the social and contextual nature of proof, offer the following relativist description: We call proof an explanation accepted by a given community at a given time [6]. An argument becomes a proof after the social act of accepting it as a proof. Many mathematics teachers believe that focusing exclusively on the logical nature of proof can be harmful to students' development. But such a narrow view leads students to focus on logical manipulations rather than forming and understanding convincing explanation why a statement is true [7]. Mathematics educators and mathematicians believe that the veracity of a statement is only one of many reasons for constructing or presenting a proof. Besides convincing, mathematics educators have proposed some alternative purpose of proof. For example:

- Explanation: By examining a proof, a reader can understand why a certain statement is true. Many mathematics educators argue that explanation should be the primary purpose of proof in mathematics classroom [8]
- Communication: The language of proof can be used to communicate and debate ideas with other students and mathematicians [9]
- Justification of a definition: One can show that a definition is adequate to capture the intuitive essence of a concept by providing that all of the concept's essential properties can be derived from the proposed definition.
- Discovery of news results: By exploring the logical consequences of definitions and an axiomatic system, new models or theories can be developed
- Developing intuition: By examining the logical entailments of a concept's definition, one can sometimes develop a conceptual and intuitive understanding of the concept that one is studying.
- Proving autonomy: Teaching students how to prove can allow them to independently construct and validate new mathematical knowledge
- Systemization: One can use proofs to organize previously disparate results into a unified whole

By organizing a system deductively, one can also uncover arguments that may be fallacious, circular or complete. Students who believe that proofs are used only to establish the certainty of mathematical statements finds proof of seemingly obvious results to be pedantic.

Why students hate proofs?: There is a considerable evidence that students leave school with negative attitudes towards mathematics. Some dislike the subject, others feel inadequate about it, and still others feel it is irrelevant in their lives. Students entering college or university are often very adept at performing algorithms and finding their way through the maze sophisticated calculations or some geometry problems based on calculations. However, they tend to have very little experience with mathematical proofs even though these are central to verifying mathematical facts and buildings corpus of reliable knowledge. It is common for students to say that they like mathematics but hate proofs. For many students proof technique is a difficult to overcome and has all of the hallmarks of a threshold concept. The ability to understand and construct proofs is transformative, both in perceiving old ideas and making new and exciting mathematics discoveries.

In many cases it appears that negative attitudes toward proofs result from certain teaching practices, the nature of the subject, and the selection of proof problems and inability of teachers to explain conceptually difficult concepts in simple terms. When introducing a proof, some teachers assumed that students already know, or familiar with the theorems that will be used in the proof construction. Others teachers instead of explaining to students the reason of moving from step A to step B, they content to use the following words:

- Based on the theorem of (Pythagoras for example)
- Using the definition of
- By inserting α into β we have
- After developing $\emptyset$ we deduce ∂

These words: based on, using the, by inserting, after and deduce are very confusing for students. As an example before writing this article, deliberately I have used the word using the definition of (X) when proving that $0! = 1!$ to my students during the mathematics lesson. Surprisingly one of my best students asked me to state the definition again. I responded are you joking? We have learned this definition just two days ago. I am sorry; Sir if I am asking you to state it means that I get lost. Get lost mean? I asked him again, I forgot this definition he replied. This example shows that we cannot tell about students' ability of memorization. Even if teacher is sure students know the theorem or definition that will be used for proof, some

students may not remember. So, it is better to always as reminder states the theorem again so that they can caught what you are presenting or proving.

The following comments were made by university students in the Department of Pure Mathematics studying to be high school mathematic teachers. They were asked to reflect on their own experiences of learning proofs in mathematics. While these comments are not necessarily representative of all students' teachers, they indicate that teacher has a large impact on attitudes toward proofs.

The Teacher:

● The teacher went too fast and did not know how to explain difficult concept to simple terms
● I had a bad teacher who passed on dislike proofs
● The teacher did not give a reason that each proof steps is correct
● Most mathematics lessons were boring and make me sleep.
● The teacher did not state the theorem involving in the proofs
● The teacher did not convince me about the necessity of the proofs

The students:

● I was too afraid to ask questions because I did not want to look stupid.
● You either have a mathematic brain or you don't

The ways teachers teach proofs in mathematics makes difference.

How the proof should be taught?: The following two methods are some examples of how to teach proofs. The first method is the modified Moore method. The modified Moore is a teaching paradigm that is based on the pedagogical techniques of the mathematician Robert Lee Moore [10]. Modified Moore method: Moore and proponents of this method believe that students will learn little about advanced mathematics by passively writing down the proofs that the professor or instructor presents on the blackboard, and will learn far more about mathematical concepts and proofs if they try to construct the proofs by themselves. Below is a brief description of this influential teaching method. In a typical class using the Moore method, the professor or instructor presents the students with the definitions of mathematical concepts and may be a few motivating examples of those concepts. After this, students are

asked to prove or disprove a set of propositions about these concepts. When a student believes that he or she has proved a proposition, that student is invited to present his or her argument on the blackboard. The teacher and the fellow students may critique the student's work or ask the student to clarify his or her argument. If everyone including the professor is convinced by the proof, the class moves to another proposition. If no student is successfully able to prove a theorem, the teacher may ask the students to prove a simpler proposition, put the proposition off to another day, or simply let the proposition go unproved. The teacher may also provide assistance to the students, but the assistance should be minimal amount necessarily for the students to construct the proof. What is critical is that the teacher never provides the students with the actual proof of a proposition. All proofs are generated by the students by themselves.

The second method is the author method called " Z. Mbaïtiga method". In this method, once the professor presents the problem to be proved on the blackboard ask students to suggest or propose the theorem that can be used to solve the problem and explain how it should be used. After the proposition of the theorem or formula to be used for solving the problem is done and even if the professor knows that the proposed theorem is false, without saying anything uses it and solves the problem, then ask the fellow students their opinion about the proof result. Many arguments will be given by the students and among these arguments the teacher should pick up two propositions: the right proposition and one similar to the one that was proposed if possible. Write them on the other side of the blackboard and ask students again to choose the right formula or theorem. When the theorem or formula is selected, the teacher uses it and solves the problem without erasing the first false result, then asks the class again if they are convinced or not. If everyone is convinced, then the professor compares the two results and explains why the first result is false. But if no student can pointed out what is wrong with the result, the professor assist the students by proposing the theorem or formula to be used and another similarly to the right proposition then put the problem to home work for the next day. During the next day proof as a problem-based learning the teacher let students solve the problem by themselves on the blackboard, and can provide only assistance. During the proof class, the teacher should focus only on the proof instead of thinking about moving to the next lesson. Because proof is a scientific language of communication and is a very important tool that can help student to defend themselves when facing a tough

problem in other subjects. The teacher should never leave the proof unproved once presented to the students.

In both Robert Lee and Z. Mbaitiga methods all the efforts are done by the students themselves with professor assistance only. But the difference between the two methods is that in Moore method, if no student is able to prove the theorem the teacher can simply let the proposition go unproved, while in Z. Mbaitiga method the proposition should never be let unproved once presented to the students.

Case study on proof of triple integral: So, which kinds of proof method are most appropriate for a lecture or class presentation? The short answer is those which lead to deep explanation of the formulas that the teacher uses for proof construction, or those which lead as quickly as possible to deep conceptual understanding.
Here an example is given from author's recentteaching of triple integral to fourth year college students (equivalent to first year university students), who have only a high school background. It uses two cases of studies. After each case study students were asked about the case study that is easy for them to understand.

Problem to be proved: By using the spherical coordinates prove that,

$$\iiint_R (x^2 + y^2 + z^2)dxdy = \frac{56\pi}{15} \qquad (5)$$

Case study 1: In this case study, the author assumed that students already have learned or understand well how to find the spherical coordinates and can easily manipulate them to solve Eq.5. The author also assumed that students have no problem at all on trigonometric formulas conversion.

Proposition 1: Let R be a space area of x, y, z and R′ the set of $(\rho, \theta, \varphi) \in [0, \infty) \times [0, 2\pi] \times [0, \pi]$ such as $(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\sin\varphi) \in R$, the area corresponding to the space ρ, θ, φ shown in Fig. 2 has a real function f: R→R such as the triple integral.
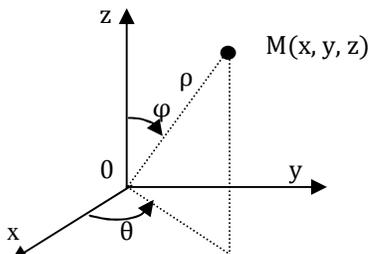

Fig.2: Spherical coordinates without details information

$\iiint_R (x + y + z)dx\,dy\,dz$ exist

So that:

$$\iiint_{R'} f(\rho\sin\varphi\cos\theta, \rho\sin\varphi\sin\theta, \rho\cos\varphi)\rho^2 \sin\varphi d\rho d\theta d\varphi$$
$$= \iiint_R (x, y, z)dxdydz \qquad (6)$$

Proposition 2: Area R′corresponding to R in spherical coordinates is:

$$R' = \begin{cases} \{(\rho,\theta,\varphi) \big| 0 \le p \le \\ \qquad 0 \le \theta \le 2\pi, \\ \varphi_{min} \le \varphi \le \pi - \varphi_{min} \} \end{cases}$$

It is easy to verify that:

$$\varphi_{min} = 1/2, \sin\varphi_{min} = 1/\sqrt{5}, \cos\varphi_{min} = 2/\sqrt{5}$$
$$\sin(\pi - \varphi_{min}) = 1/\sqrt{5}, \cos(\pi - \varphi_{min}) = -2/\sqrt{5}$$

We used the fact that $\rho\sin\varphi = 1$ for the point on the vertical cylindrical edge of our area.

Proof 1: From proposition 1 and 2, Eq. 5 becomes:

$$\iiint_R (x^2 + y^2, z^2)dxdydz$$
$$= \iiint_R p^2 . \rho^2 \sin p d\theta d\varphi$$
$$= \int_0^{2\pi} (\int_{\varphi_{min}}^{\pi-\varphi_{min}} (\int_0^{1/\sin\varphi} \rho^4 \sin\varphi d\rho)d\varphi)d\theta$$
$$= \int_0^{2\pi} (\int_{\varphi_{min}}^{\pi-\varphi_{min}} (\frac{\rho^5}{5} \Big]_{p=0}^{p=1/\sin\varphi} \sin\varphi d\varphi)d\theta$$
$$= \int_0^{2\pi} (\int_{\varphi_{min}}^{\pi-\varphi_{min}} \frac{1}{5\sin^4\varphi}d\varphi)d\theta \qquad (7)$$
$$= \int_0^{2\pi} (\frac{-\cos\varphi}{3\sin^3\varphi} - 2\frac{\cos\varphi}{3\sin\varphi} \Big]_{\varphi=\varphi_{min}}^{\varphi=\pi-\varphi_{min}} d\theta$$
$$= \frac{28}{15}\int_0^{2\pi} d\theta = \frac{28}{15}(\theta]_{\theta=0}^{\theta=2\pi} = \frac{56\pi}{15}$$

Hence the proof of Eq. 5 is completed.

Case study 2: In this case study the author assumed that, students have learned the spherical coordinates but did not understand how to use them and have also some difficulties on trigonometric formulas conversions and have limited or no experience with proof construction. Therefore more details are required.

Proposition 3: Spherical coordinates consist of the following three quantities:

Radius: $\rho = \overrightarrow{OMp}$

Azimuth: $\theta = (\overrightarrow{U_x}, \overrightarrow{OH})$

Colatitude: $\varphi = (\overrightarrow{U_z}, \overrightarrow{U_p}) = 90° - \delta$ (δ = latitude)

ρ = Distance from the origin to the point M and will require $\rho \geq 0$
θ = The same angle we see in polar-cylindrical coordinates. It is the angle between the positive x-axis and the line above denoted by r which is also the same r as in polar-cylindrical coordinates shown in Fig.3. There is no restriction on θ. That is, $0 \leq \theta \leq 2\pi$
φ = Angle between the positive z-axis and the line from the origin to the point M, with $0 \leq \varphi \leq \pi$

In summary, ρ is the distance from the origin to the point M, φ is the angle that we need to rotate down from the positive z-axis to get the point M and θ is how much we need to rotate around the z-axis to get to the point M. Now we should first derive some conversion formulas. Let's first start with a point in spherical coordinates and ask what the cylindrical coordinates of the point are. So, we know (ρ, θ, φ) and what to find r, θ, z . Of course we really only need to find r and z since θ is the same angle in both coordinates systems. We will be able to do all of our work by looking at the right angle shown in Fig. 3.With little geometry using the triangle represented by OPM we see that the angle between z and ρ is φ and we can see that:

$$z = \rho\cos\varphi \qquad (8)$$
$$r = \rho\sin\varphi$$

And there are exactly the formulas we were looking for. So given a point in spherical coordinates the cylindrical coordinates of the point will be:

$$r = \rho\sin\varphi$$
$$\Theta = \theta \qquad (9)$$
$$z = \rho\cos\varphi$$

Next, let's find the Cartesian coordinates of the same point. To do this we will start with the cylindrical conversion formulas, Fig. 4.

The conversions for x and y are the same conversions that we used back in when we were looking at polar coordinates. So if we have a point in
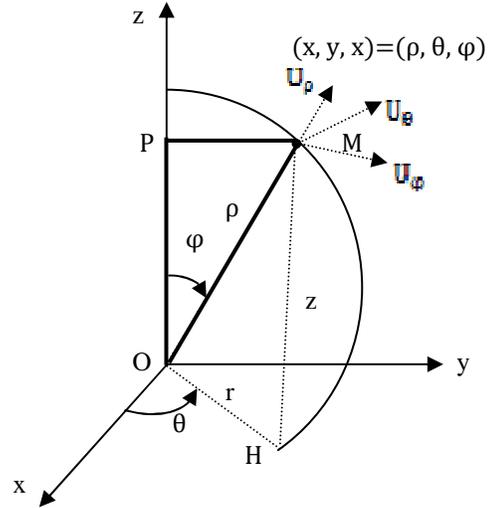

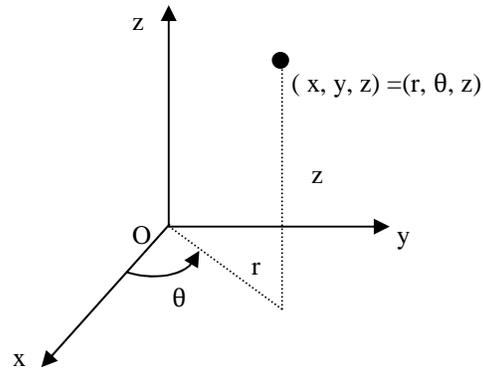Fig.3: Spherical coordinates with details information

Fig.4: Cylindrical coordinates

cylindrical coordinates the Cartesian coordinates can be found by using the following conversions:

$$x = r\cos\theta$$
$$y = r\sin\theta \qquad (10)$$
$$z = z$$

The third equation is just an acknowledgement that the z-coordinate of a point in Cartesian and polar coordinates is the same. Now all that we need to do is to use Eq. 9 for r and z to get:

$$\begin{cases} x = \rho\sin\varphi\cos\theta \\ y = \rho\sin\varphi\cos\theta \\ z = \rho\cos\varphi \end{cases} \text{and} \begin{cases} x = \sqrt{x^2 + y^2 + z^2} \\ \theta = \arctan\dfrac{y}{x} \\ \varphi = \arctan(\dfrac{\sqrt{x^2 + y^2}}{z}) \end{cases} \qquad (11)$$

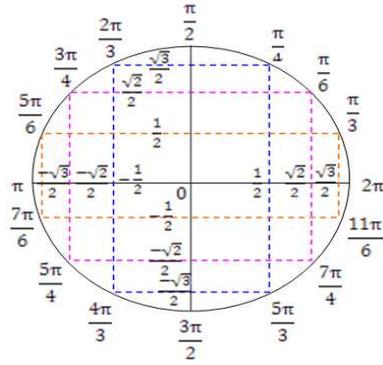Also note that since we know that:

Fig. 5: Trigonometric circle

$$r^2 = x^2 + y^2 \qquad (12)$$

We get:

$$\begin{cases} \rho^2 = x^2 + y^2 + z^2 \\ \text{and} \\ dx\,dy\,dz = \rho^2 \sin\varphi\,d\rho\,d\theta\,d\varphi \end{cases} \qquad (13)$$

Proposition 4: Area R' corresponding to R in spherical coordinates is:

$$R' = \begin{cases} \{(\rho,\theta,\varphi) | 0 \le 0 \le 1/\sin\varphi \\ \quad 0 \le \theta \le 2\pi, \\ \quad \varphi_{min} \le \varphi \le \pi - \varphi_{min}\} \end{cases} \qquad (14)$$

The researcher has used the trigonometric circle shown in Fig. 5 to explain how the values of Eq. 14 have been obtained to students by considering the fact that $p\sin\varphi = 1$ for the point on the vertical cylindrical edge of our area. But for the sake of brevity the author refrain from giving the details.

$$\begin{cases} \varphi_{min} = 1/2 \\ \sin\varphi_{min} = 1/\sqrt{5}, \\ \cos\varphi_{min} = 2/\sqrt{5} \end{cases} \begin{cases} \sin(\pi - \varphi_{min}) = 1/\sqrt{5} \\ \cos(\pi - \varphi_{min}) = -2/\sqrt{5} \end{cases} \qquad (14)$$

Proof 2: From proposition 3 and 4 Eq. 5 becomes:

$$\iiint_R (x^2 + x^2 + x^2)\,dx\,dy\,dz$$
$$= \iiint_R \rho^2 \cdot \rho^2 \sin p\,dp\,d\theta\,d\varphi$$
$$= \iiint_0^{2\pi} (\int_{\varphi_{min}}^{\pi-\varphi_{min}} (\int_0^{1/\sin\varphi} \rho^4 \sin\varphi\,dp)\,d\varphi)\,d\theta \qquad (15)$$
$$= \int_0^{2\pi} \int_{\varphi_{min}}^{\pi-\varphi_{min}} (\frac{\rho^5}{5}]_{p=0}^{p=1/\sin\varphi} \sin\varphi\,d\varphi)\,d\theta$$
$$= \int_0^{2\pi} \int_{\varphi_{min}}^{\pi-\varphi_{min}} \frac{1}{5\sin^4\varphi}\,d\varphi)\,d\theta$$

Now let's find the primitive of:

$$\int \frac{1}{\sin^4\varphi}\,d\varphi$$

We know that: $\cot\varphi = \dfrac{\cos\varphi}{\sin\varphi}$ and its derivation is:

$$\frac{d}{d\varphi}(\cot an\varphi) = \frac{d}{d\varphi}(\frac{\cos\varphi}{\sin\varphi}) = \frac{-1}{\sin^2\varphi} \qquad (16)$$

Then:

$$\int \frac{1}{\sin^4\varphi}\,d\varphi \int \frac{1}{\sin^2\varphi} \frac{d}{d\varphi}(\frac{-\cos\varphi}{\sin\varphi})\,d\varphi$$
$$= \frac{1}{\sin^2\varphi}(\frac{-\cos\varphi}{\sin\varphi}) - \int (\frac{-\cos\varphi}{\sin\varphi} - \frac{(-2)\cos\varphi}{\sin^3\varphi})\,d\varphi \qquad (17)$$
$$= (\frac{-\cos\varphi}{\sin^3\kappa}) - 2\int \frac{\cos^2\varphi}{\sin^4\varphi}\,d\varphi$$

Replacing $\cos^2\varphi$ by $1-\sin^2\varphi$ into the second term of Eq. 17, since we know that $\cos^2\varphi + \sin^2\varphi = 1$
We have:

$$= (\frac{-\cos\varphi}{\sin^3\varphi}) - 2\int (\frac{(1-\sin^2\varphi)}{\sin^4\varphi})\,d\varphi$$
$$= (\frac{-\cos\varphi}{\sin^3\kappa}) - 2\int \frac{1}{\sin^4\varphi}\,d\varphi + 2\int \frac{\sin^2\varphi}{\sin^4\varphi}\,d\varphi$$
$$= (\frac{-\cos\varphi}{\sin^3\varphi}) - 2\int \frac{1}{\sin^4\varphi} + 2\int \frac{1}{\sin^2\varphi}\,d\varphi$$

Hence:

$$3\int \frac{1}{\sin^4\varphi}\,d\varphi = \frac{-\cos\varphi}{\sin^3} + 2\int \frac{1}{\sin^2\varphi}\,d\varphi$$
$$\int \frac{1}{\sin^4\varphi}\,d\varphi = \frac{-\cos\varphi}{3\sin^3\varphi} - 2\int \frac{\cos\varphi}{3\sin\varphi} + c \qquad (18)$$

Using Eq. 18 our Eq. 15 becomes:

$$= \frac{1}{5}\int_0^{2\pi} \left(\frac{-\cos\varphi}{3\sin^3\varphi} - 2\frac{\cos\varphi}{3\sin\varphi}\right]_{\varphi=\varphi_{min}}^{\varphi=\pi-\varphi_{min}} d\theta$$
$$= \frac{1}{5}\int_0^{2\pi} \left(\frac{-1(-2/\sqrt{5})}{3(1/\sqrt{5})^3} - \frac{-2(-2/\sqrt{5})}{3(1/\sqrt{5})}\right)$$
$$- \left(\frac{-1(2/\sqrt{5})}{3(1/\sqrt{5})^3} - \frac{2(2/\sqrt{5})}{3(1/\sqrt{5})}\right) d\theta$$
$$= \frac{28}{15}\int_0^{2\pi} d\theta = \frac{28}{15}[\theta]_{\theta=0}^{\theta=2\pi} - \frac{56\pi}{15}$$

RESULTS AND DISCUSSION

Very few teachers would cite assurance of truth as the sole reason for teaching proof in the classroom, of course. Proof is also a method of communicating results to others in a clear and fairly conventional form. This purpose relatively straightforward; a good proof show in details the problem to be proved follows from other already-known facts by a chain of good reasoning. The way teacher teach proofs makes difference and can have an impact on students. Let's see the results of the survey conducted on four year college students shown in Fig. 6-8 on case study 1 and 2. The class was divided in to 3 groups as follows: High level students which consist of 12 students, average level students which consist of 20 students and low level students which consist of 13 students respectively. Each group was asked on which of the four propositions and proofs they have no problem, some problems understanding it or simply cannot understand.

High level students: Case study 1, 2: In Fig. 6a on 12 students who responded. 4, 10 and 4 students said that they have no problem at all understanding the proposition 1, 2 and proof 1.

6, 2 and 7 have some problems, while in Fig. 6b all 12 students who responded said with interesting comments that they have no problem at all understanding the proposition 3, 4 and proof 2.

Average level students: Case study 1, 2: Concerning the average level students' results, in Fig. 7. On 19 students who responded, Fig. 7a: 2, 4 and 1 students said that they have no problem at all understanding the proposition 1, 2 and proof 1.

10, 8 and 2 have some problems, 7, 8 and 17 cannot simply understand. There was one student who did not respond for proposition1. While in Fig. 7b, 18, 19 and 18 have no problem for proposition 3, 4 and proof 2 and only 2, 1 and 2 have some problems.

Low level students: Case study 1, 2: Regarding the low level results shown in Fig. 8a and b. On 13 students who responded, no student have been found to have understood the proposition1, 2 and proof 1. 1, 4 and 1 have some problems, but 12, 10 and 11 cannot simply understand; as there was one unresponded for proof 2. While in Fig. 8b the result shows that 9, 11 and 10 have no problem understanding the proposition 3, 4 and proof 2 and only 3, 2 and 3 have some problems. The result of this survey shows that most students have no problem understanding the proof as well as theorems involving in the proof construction with more details.
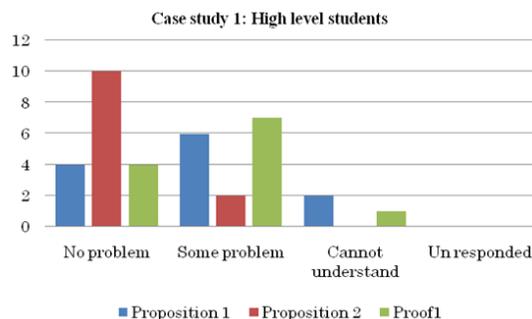

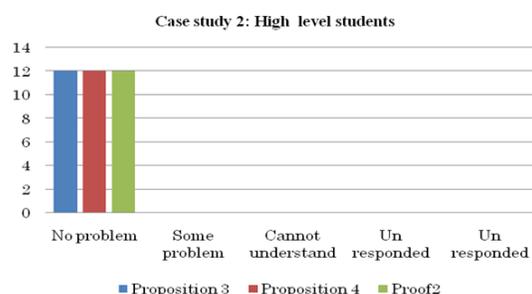Fig. 6a: Survey result of case study 1

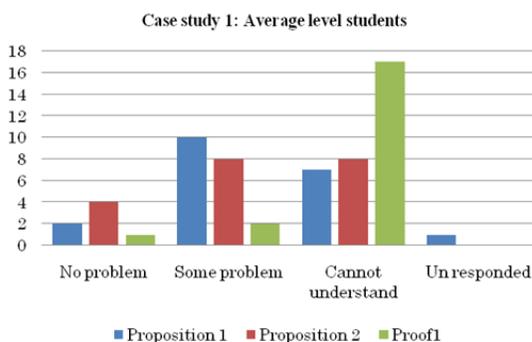

Fig. 6b: Survey result of case study 2

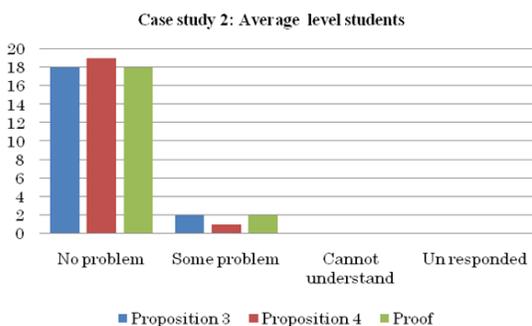Fig. 7a: Survey result of case study 1

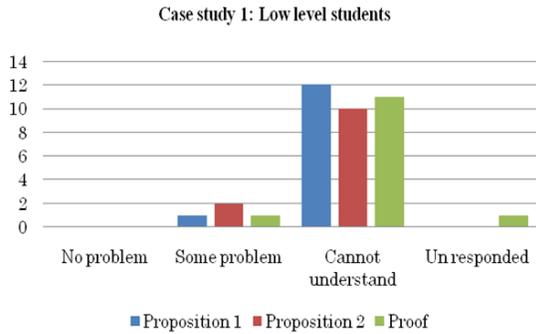Fig. 7b: Survey result of case study 2
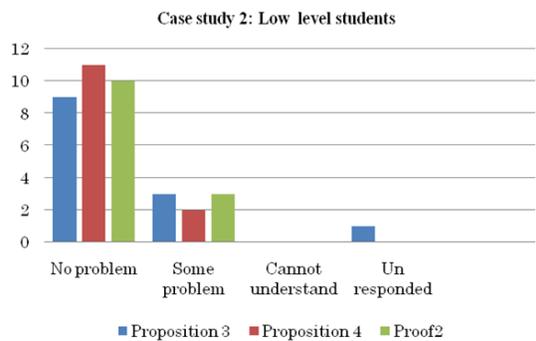
Fig. 8a: Survey result of case study 1

Fig. 8b: Survey result of case study 1

That is, the way teachers explain things makes difference and can encourage students to like proofs or simply like mathematics.

How can students do proof in mathematics?: An important part of proofs is to understand the proofs generated by other people. Students need to learn how to read proofs, understanding the various component steps and the logical relationships between them. Understanding a proof depends on the background and assumptions being made by the person doing proof. When constructing a proof it is important to understand that once something is proved, there are no counterexamples that contradict the proof. A proof is true in all circumstances under the conditions by which it was constructed. Thus, not only does understanding a proof constitute being able to recognize what is and what is not a proof, it also must include recognizing that a proof means that there are no exceptions from the proof. Writing a mathematic proof is the hardest part of mathematic, but to overcome this difficulty, there are some guidelines to follow. These guidelines can help students to erase the doubt from the validity of his proof: Below is one of the ways students can do proof:

- Write what was given as well as what is needed to be proven. Here it shows that you will start with what is given, use theorems, formulas or other results you know to be true
- Ask yourself questions as you move along. Why is this so? And is there any way this can be false? These questions will be asked by your professor to check if you understand what you have proved. Back up every statement with a reason and justify your process
- Ask your professor or classmate if you get lost or have some problems. It is good to ask questions every now and then, it is a part of the learning process. Do not thing that asking a classmate a question you will look stupid. This kind of thinking is a big mistake
- Make sure your proof is step-by-step. It needs to flow from one statement to the other, with support for each statement, so that there is no doubt the validity of your proof. It should be well constructed. Orderly, systematic and with properly paced progress

Finally designate the end of your proof. There are several ways, whose two are proposed below:

- A filled-in square at the end of the proof
- If you are not sure whether your proof is correct, write a few sentences saying what your conclusion was and why it is significant

Remember, proof that have been given by your professor, was already proven, which means that is usually true. If you came to a conclusion that is different from what was to prove, then more than likely you messed up somewhere. Just go back and review each step. Writing multiple drafts for your proof is not uncommon. Some important information on the proof can be found in[11] where the author developed in details the proof in school and proof and problem solving.

CONCLUSION

Mathematics educators in college and university aims at providing a certain level of understanding of mathematic and mathematical methods, most of students will not continue their studies of mathematics, but they will have to apply their knowledge of mathematics in such fields as sciences, business there are some guidelines to follow. These guidelines can administration and engineering. Therefore mathematics

teachers have to be very careful about the selection of proofs to include when introducing topics and filtering out certain details which can obscure important ideas. Indeed the word proof is often equated with obfuscation. A poorly presented proof even if meticulously prepared, can be frustrating and wasteful in terms of time and effort in concentration and it is common for students to get lost. In many cases it appears that negative attitudes toward proofs result from certain teaching practices, the nature of the subject and the selection of proof problems and inability of teachers to explain conceptually difficult concepts in simple terms. The objective of this study is not to take mathematics professors or educators responsible of the dislike of proofs by students, but instead to let them know that students rely on them so they have to help them to erase the mystery behind the proof.

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