Journal of Mathematics and Statistics

Original Research Paper

Some Generalized Fixed Point Theorems of Contraction Type Mappings in Quasi Metric Spaces

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Article history
Received: 31-05-2017
Revised: 29-08-2017
Accepted: 6-10-2017

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Abstract: This paper adjusts conditions on new defined given the name $\{a, d, c; r\}$ ctype of contraction mappings on complete quasi metric space, confirms that there is only one fixed of any of these mappings with these adjustments, extends and generalizes results given in some previous research papers, and then builds a convergence theorem for a sequence of fixed points of $\{S_n\}_{n\in N}; \{w_n^*\}_{n\in N}$ to the unique fixed of S; w, provided that $\lim_{n\to\infty} S_n(w) = S(w)$.

Keywords: Fixed Point Theorem, Generalized $\{a,b,c\}$- Contraction and Nonexpansive Mappings

Introduction

Preliminaries

Generalizations of Banach contraction principle (Banach, 1922) have been established in various settings. Alber and Guerre-Delabriere (1997) introduced the definition of weakly contractive mappings and they proved some fixed point theorems in case of Hilbert spaces, Rhoades in (Rhoades, 1977; 2001) give extended results of (Alber and Guerre-Delabriere, 1997) to complete metric spaces, and more discussed concepts shown in (Hardy and Rogers, 1973); (Sehie, 1980), and (Sahar Mohamed Ali Abou Bakr, 2013).

In this paper X will be a nonempty set and S be mapping from X into X, we have:

Definition 1

If X is normed space, we have:

(1) If there are reals a, b, $c \in [0, 1]$ with $a + b + c < 1$ such that:

$$\| S(u) - S(v) \| \le a \| u - v \| + b \| v - S(v) \| + c \| u - S(u) \| \; \forall u, v \in X$$

then S is $\{a, b, c\}$ generalized contraction on X.

(2) If there are reals a, b, $c\in [0, 1]$ with $a + b + c = 1$ such that:

$$\| S(u) - S(v) \| \le a \| u - v \| + b \| v - S(v) \| + c \| u - S(u) \| \; \forall u, v \in X$$

then S is $\{a, b, c\}$ generalized nonexpansive.

(3) If there are reals a, b, $c\in[0, 1]$ with $0 \le c < \frac{1}{2}$ and $a + b + c < 1$ such that:

$$\| S(u) - S(v) \| \le a \| u - v \| + b \| v - S(v) \| + c \max\{\| u - S(u) \|, \| x - S(y) \|\}$$

for all u, $v\in X$, then S is $\{a, b, c\}$-ctype mapping.

(4) If there are reals a, b, $c\in[0, 1]$ with $0 < a < 1, 0 < b, 0 \le c < \frac{1}{2}$ and $a + b + c = 1$ such that:

$$\| S(u) - S(v) \| \le a \| u - v \| + b \| v - S(v) \| + c \max\{\| u - S(u) \|, \| u - S(v) \|\}$$

for all u, $v\in X$, then S is $\{a, b, c\}$-ntype mapping.

El-Shobaky *et al.* (2007) proved that there is only one fixed point (*fp*) of $\{a, b, c\}$-generalized contraction mapping defined on closed and convex subset of weakly Cauchy normed space X. The existence of only one *fp* of $\{a, b, c\}$-generalized non expansive mapping when C contains contraction point proved in (Sahar Mohamed Ali Abou Bakr, 2009). Fixed point theorems established for $\{a, b, c\}$-ctype and ntype in (Sahar Mohamed Ali Abou Bakr, 2013). Mainly; we are interested in:

Theorem 1

Sahar Mohamed Ali Abou Bakr (2013) Let X be a Banach space X, S be $\{a, b, c\}$-ctype mapping, then S has only one *fp*. Moreover, for any $u\in X$, the iterated sequence $\{S^n(u)\}_{n\in N}$ is convergent to the *fp* of S.

DOI: 10.3844/jmssp.2017.319.324

In case of quasi-metric spaces, we have:

Definition 2 Sahar Mohamed Ali Abou Bakr *(2013)*

(1) Let r be a number; $r \geq 1$ and q be a mapping $q: X \to R+$. Then (X, q) is quasi-metric space iff the following:
   (a) $q(u, v) = 0$ iff $u = v$.
   (b) $q(u, v) = q(v, u)$ for all $u, v \in X$.
   (c) $q(u, v) \leq r[q(u, w) + q(w, v)]$ for all $u, v, w \in X$

(2) Let (X, q) be a quasi-metric space and $\{v_n\}_{n\in N}$ be a sequence in X. Then $\{v_n\}_{n\in N}$ is
   (a) Cauchy iff for every $\epsilon > 0$ there is $n_0(\epsilon) \in N$ such that:

$$q(v_n, v_m) < \epsilon \ \forall_{n,m} \geq n_0(\epsilon).$$

   (b) Convergent to v iff for every $\epsilon > 0$ there is $n_0(\epsilon) \in N$ such that:

$$q(v_n, v) < \epsilon \ \forall_n \geq n_0(\epsilon).$$

Remark

It is clear that every metric space is quasi-metric space (X, q) with $r = 1$. There are many examples in the literatures of quasi metric spaces which are not metric.

Now, for the purpose of the first result of this paper we define the contraction type of mappings, namely $\{a, b, c; r\}$-contraction type.

Definition 3

Let (X, q) be a quasi-metric space. Then S that satisfy $q(S(u), S(v)) \leq aq(u, v) + d\, q(v, S(v)) + c \max\{q(u, S(u)), q(u, S(v))\}$ for all $u, v \in X$ and some real numbers $a, d, c \in [0, 1]$ where $a+c < 1$ and $r(a+d)+c < 1$ is called $\{a, d, c; r\}$-ctype mapping.

Remark

Noticed the following:

(1) If (X, q) is a quasi-metric space, S satisfies $q(S(u), S(v)) \leq aq(u, v) + d\, q(v, S(v)) + cq(u, S(u)) + eq(u, S(v))$ for all $u, v \in X$ and for some a, d, c and $e \in [0, 1]$, $0 \leq a + (c + e) < 1$, and $r(a + d) + c + e < 1$, then S is $\{a, d, c + e; r\}$ *g-ctype*
(2) If $r = 1$ the definitions will be reduced to the definition of $\{a, d, c\}$ctype mappings (Ali, 2013).
(3) The class of $\{a, d, c; r\}$-ctype mappings is wider than the class of contraction mappings.

Main Results

We start with some basic lemmas:

Lemma 1

Let (X, q) be quasi-metric and $\{v_n\}_{n\in N}$ be in X such that:

$$q(v_{n+2}, v_{n+1}) \leq r_0\, q(v_{n+1}, v_n), n = 0,1,2,... \tag{2.1}$$

for some positive real number r_0 with r $r_0 < 1$. Then $\{v_n\}_{n\in N}$ is Cauchy.

Proof

We have the following:

$$\begin{aligned} q(v_{n+2}, v_{n+1}) &\leq r_0\, q(v_{n+1}, v_n) \\ &\leq r_0 r_0\, q(v_n, v_{n-1}) = r_0^2 q(v_n, v_{n-1}) \end{aligned}$$

continuing in this process gives:

$$q(v_{n+2}, v_{n+1}) \leq r_0^{n+1} q(v_1, v_0) \tag{2.2}$$

Now; let $n, m \in N$ be such that $m > n$. We have:

$$\begin{aligned} q(v_n, v_m) &\leq r\left[q\left(v_n, v_{n+1}\right) + q\left(v_{n+1}, v_m\right)\right] \\ &\leq r\left[q\left(v_n, v_{n+1}\right) + r\left\{q\left(v_{n+1}, v_{n+2}\right) + q\left(v_{n+2}, v_m\right)\right\}\right] \\ &\leq rq\left(v_n, v_{n+1}\right) r^2 q\left(v_{n+1}, v_{n+2}\right) + \\ &\quad + r^2\left[r\left\{q\left(v_{n+2}, v_{n+3}\right)\right\} + q\left(v_{n+3}, v_m\right)\right] \\ &\leq rq\left(v_n, v_{n+1}\right) r^2 q\left(v_{n+1}, v_{n+2}\right) + r^3 q\left(v_{n+2}, v_{n+3}\right) + \\ &\quad + \ldots + r^{m-n} q(v_{m-1}, v_m) \end{aligned}$$

Back to inequalities (2.2), we get:

$$\begin{aligned} q(v_n, v_m) &\leq r r_0^n q\left(v_1, v_0\right) + r^2 r_0^{n+1} q\left(v_1, v_0\right) \\ &\quad + r^3 r_0^{n+2} q\left(v_1, v_0\right) + \ldots \\ &\quad + r^{m-n} r_0^{n+(m-1-n)} q(v_1, v_0) \\ &= r r_0^n \begin{bmatrix} q\left(v_1, v_0\right) + r r_0\, q\left(v_1, v_0\right) \\ +(r r_0)^2 q\left(v_1, v_0\right) + \ldots \\ +(r r_0)^{(m-1-n)} q(v_1, v_0) \end{bmatrix} \\ &= r r_0^n q\left(v_1, v_0\right)\left[\frac{1-(r r_0)^{m-1-n}}{1 - r r_0}\right] \\ &\leq \left[\frac{r r_0^n}{1 - r r_0}\right] q\left(v_1, v_0\right) \end{aligned}$$

Clearly, $r_0 < 1$, taking the limit as $n \to \infty$ completes the proof.

Lemma 2

Let (X, q) be quasi-metric space and S satisfy $q(S(u), S(v)) \leq aq(u, v) + d\, q(v, S(v)) + c \max\{q(u, S(u), q(u, S(v))\}$ for all $u, v \in X$ and for some $a, d, c \in [0, 1]$ with $c \neq 1$. Then for any $u \in X$, $\{S^n(u)\}n \in N$ satisfies:

$$q(S^{n+1}(u), S^n(u)) \leq \left(\frac{a+d}{1-c}\right) q(S^n(u), S^{n-1}(u)) \tag{2.3}$$

$$q(S^{n+1}(u), S^n(u)) \leq \left(\frac{a+d}{1-c}\right)^n q(S(u), u) \tag{2.4}$$

Proof

Let $u \in X$, we have:

$$\begin{aligned} & q(S^{n+1}(u), S^n(u)) \\ & = q(S(S^n(u)), S(S^{n-1}(u)) \\ & \leq aq(S^n(u), S^{n-1}(u)) + dq(S^{n-1}(u), S(S^{n-1}(u))) + \\ & + c \max\left\{q\left(S^n(u), S\left(S^n(u)\right)\right), q\left(S^n(u), S\left(S^{n-1}(u)\right)\right)\right\} \\ & \leq (a+d) q(S^n(u), S^{n-1}(u)) + cq(S^{n+1}(u), S^n(u)) \end{aligned}$$

This shows that:

$$(1-c) q(S^{n+1}(u), S^n(u)) \leq (a+d) q(S^n(u), S^{n-1}(u))$$

And hence (2.3) is proved. Continuing this inductive process proves (2.4).

We also have:

Theorem 2

Let (X, q) be a Complete quasi-metric space, S be $\{a, d, c, r\}$-ctype mapping. Then S has only one *fp*; y. Moreover, for any $u \in X$, $\{S^n(u)\}_{n \in N}$ is converging to y.

Proof

Let $u \in X$ be an arbitrarily chosen element, using Lemma (2) insures that:

$$\begin{aligned} & q(S^{n+1}(u), S^n(u)) \leq \left(\frac{a+d}{1-c}\right) \\ & q(S^n(u), S^{n-1}(u)), n = 0, 1, 2, \ldots \end{aligned}$$

Since S is $\{a, d, c, r\}$-ctype, we have $r(a + d) + c < 1$, consequently $r\left[\frac{a+d}{1-c}\right] < 1$. Taking $r_0 = \left[\frac{a+d}{1-c}\right]$ insures that the inequalities (2.3) of Lemma (1) are satisfied, then we have:

$$\lim_{n \to \infty} q(S^{n+1}(u), S^n(u))) = 0 \tag{2.5}$$

and $\{S^n(u)\}_{n \in N}$ is Cauchy. Since X is complete, $\{S^n(u)\}_{n \in N}$ is converging to some element $y \in X$:

$$\lim_{n \to \infty} q(S^n(u), y) = 0 \tag{2.6}$$

On the other side:

$$\begin{aligned} & q\left(S(y), S^{n+1}(u)\right) \geq aq\left(y, S^n(u)\right) \\ & + d\, q\left(S^n(u), Sn+1(u)\right) \\ & + c \max\left\{q(y, S(y)), q(y, S^{n+1}(u))\right\} \end{aligned}$$

Accordingly, we have:

$$\begin{aligned} & q\left(S(y), y\right) \leq rq\left(S(y), S^{n+1}(u)\right) + rq\left(S^{n+1}(u), y\right) \\ & \leq r\left[\begin{aligned} & aq\left(y, S^n(u)\right) + dq\left(S^n(u), S^{n+1}(u)\right) \\ & + c \max\left\{q\left(y, S(y)\right), q\left(y, S^{n+1}(u)\right)\right\} \end{aligned}\right] + rq\left(S^{n+1}(u), y\right) \end{aligned}$$

Taking the limit as $n \to \infty$ with the use of (2.5) and (2.6) prove:

$$q\left(S(y), y\right) \leq cq\left(y, S(y)\right)$$

If $q(S(y), y) \neq 0$, then $1 \leq c$, this yields $q(S(y), y) = 0$, consequently $S(y) = y$.

Finally to prove that there is only one of such a point, let y and z be two distinct points of S such that $S(y) = y$ and $S(z) = z$. Then the following strict inequality gives an obvious contradiction:

$$\begin{aligned} q\left(y, z\right) & = q\left(S(y), S(z)\right) \\ & \leq aq\left(y, z\right) + d\, q\left(z, S(z)\right) \\ & + c \max\left\{q\left(y, S(y)\right), q\left(y, S(z)\right)\right\} \\ & = (a + c) q(y; z) \\ & < q(y; z). \end{aligned}$$

Corollary 1

Let (X, q) be complete and S satisfy:

$$\begin{aligned} q\left(S(u), S(v)\right) & \leq aq\left(u, v\right) + d\, q\left(v, S(v)\right) \\ & + cq\left(u, S(u)\right) + eq\left(u, S(v)\right) \end{aligned}$$

for all $u, v \in X$ *and* for some a, d, c and $e \in [0, 1]$, $0 \leq a + c + e < 1$, and $r(a + d) + c + e < 1$. Then S has only one *fp*.

Proof

Let S satisfy $q(S(u), S(v)) \leq aq(u, v) + d\, q(v, S(v)) + cq(u, S(u)) + eq(u, S(v))$ for all $u, v \in X$ and for some real numbers a, d, c and $e \in [0, 1]$, $r(a + d) + c + e < 1$. Then

$q(S(u), S(v)) \le aq(u, v) + d\, q(v, S(v)) + (c + e)$ max $\{q(u, S(u)), q(u, S(v))\}$ for all $u, v \in C$, since $r(a + d)+(c + e) <1$, S is $\{a, d, c + e\}$-ctype mapping, using Theorem (1) proves that S has only one *fp*.

In some situations it is difficult to determine the fixed point of $\{a, d, c, r\}$ types of mapping, approximating such a fixed point is va-porable tool. For this direction we have the following result.

Theorem 3

If $\{S_n\}_{n\in N}$ and S are $\{a, d, c, r\}$-ctype of mappings on the complete quasi metric space (X, q), wn is the unique fixed of each S_n, $S_n(w_n^*) = w_n^*$ and $\{S_n(w_n^*)\}_{n\in N}$ converges to w^*,

$$\lim_{n\to\infty} q(S_n(w^*), S(w^*)) = 0,$$

where, w^* is the unique fixed of S; $S(w^*) = w^*$, then $\{w_n^*\}_{n\in N}$ converges to w^*:

$$\lim_{n\to\infty} q\left(w_n^*, w_n^*\right) = 0$$

Proof

We have:

$$q(w_n^*, w^*) = q\left(S_n(w_n^*), S(w^*)\right) \le r\left[q\left(S_n(w_n^*), S(w_n^*)\right) + q\left(S(w_n^*), S(w^*)\right)\right] \quad (2.7)$$

Because of contraction condition we have:

$$\begin{aligned} q\left(S(w_n^*), S(w^*)\right) &\le aq(w_n^*, w^*) + dq\left(w_n^*, S(w_n^*)\right) + \\ &+ c\max\left\{q(w^*, S(w^*)), q\left(w^*, S(w_n^*)\right)\right\} \\ &\le aq(w_n^*, w^*) + dq\left(S_n(w_n^*), S(w_n^*)\right) + \\ &+ c\max\left\{q(w^*, w^*), q\left(w^*, S(w_n^*)\right)\right) \\ &\le aq(w_n^*, w^*) + dq\left(S_n(w_n^*), S(w_n^*)\right) + \\ &+ cq\left(S(w^*), S(w_n^*)\right) \end{aligned} \quad (2.8)$$

Inequalities (2.8) shows that:

$$q\left(S(w_n^*), S(w^*)\right) \le \frac{1}{1-c}\left[aq(w_n^*, w^*) + dq\left(S_n(w_n^*), S((w_n^*)\right)\right] \quad (2.9)$$

Using (2.9) in (2.7) gives:

$$q(w_n^*, w^*) \le rq\left(S_n(w_n^*), S(w_n^*)\right) + \frac{r}{1-c}\begin{bmatrix} aq\left(w_n^*, w^*\right) \\ +dq\left(S_n(w_n^*), S(w_n^*)\right) \end{bmatrix} \quad (2.10)$$

Inequalities (2.10) shows that:

$$q(w_n^*, w^*) \le \left|\frac{1-c}{1-c-ra}\right| r\left[1 + \frac{d}{1-c}\right] q,\left(S_n(w_n^*), S(w_n^*)\right)$$

Hence:

$$q(w_n^*, w^*) \le \left|\frac{(1-c+d)r}{1-c-ra}\right| q,\left(S_n(w_n^*), S(w_n^*)\right) \quad (2.11)$$

Since:

$$\begin{aligned} &q(S_n(w_n^*), S(w^*{}_n)) \\ &\le r[q(S_n(w_n^*), S_n(w^*)) + q(S_n(w^*), S(w_n^*))] \\ &\le rq(S_n(w_n^*), S_n(w^*)) + \\ &+ r^2 q(S_n(w^*), S(w^*)) + r^2 q(S(w^*), S(w^*{}_n)) \end{aligned} \quad (2.12)$$

Using (2.9) in (2.12):

$$\begin{aligned} &q\left(S_n(w_n^*), S(w_n^*)\right) \\ &\le rq\left(S_n(w_n^*), S_n(w^*) + r^2 q\left(S_n(w^*), S(w^*)\right)\right. \\ &+ \frac{r^2}{1-c}\left[aq(w_n^*, w^*) + dq\left(S_n(w_n^*), S(w_n^*)\right)\right] \end{aligned} \quad (2.13)$$

Inequalities (2.13) proves:

$$q\left(S_n(w_n^*), S(w_n^*)\right) \le \left[\frac{1-c}{1-c-r^2 d}\right] \begin{bmatrix} rq\left(S_n(w_n^*), S_n(w^*)\right) + \\ +r^2 q\left(S_n(w^*), S(w^*)\right) + \frac{ar^2}{1-c} q(w_n^*, w^*) \end{bmatrix} \quad (2.14)$$

On the other hand:

$$\begin{aligned} q\left(S_n(w_n^*), S_n(w^*)\right) &\le aq(w_n^*, w^*) + dq\left(w_n^*, S_n(w^*)\right) + \\ &+ c\max\left\{q(w_n^*, S_n(w_n^*)), q\left(w_n^*, S_n(w^*)\right)\right\} \\ &\le aq(w_n^*, w^*) + dq\left(S(w^*), S_n(w^*)\right) + \\ &+ c\max\left\{q(w_n^*, w_n^*), q\left(S_n(w_n^*), S_n(w^*)\right)\right\} \\ &\le aq(w_n^*, w^*) + dq\left(S(w^*), S_n(w^*)\right) + \\ &+ cq\left(S_n(w_n^*), S_n(w^*)\right) \\ &+ cq\left(S(w^*), S(w^*)\right) \end{aligned} \quad (2.15)$$

Consequently:

DOI: 10.3844/jmssp.2017.319.324

$$q\left(S_n(w_n^*), S_n(w^*)\right) \le \left[\frac{1}{1-c}\right] \left[aq(w_n^*, w^*) + dq\left(S(w^*), S_n(w^*)\right)\right] \quad (2.16)$$

Using (2.16) in (2.14):

$$q(S_n(w_n^*), S(w_n^*)) \le [\frac{1-c}{1-c-r^2 d}] \left\{ \begin{array}{l} (\frac{r}{1-c})[aq(w_n^*, w^*) + d\, q(S(w^*), S_n(w^*))] \\ + r^2 q(S_n(w^*), S(w^*)) + (\frac{ar^2}{1-c}) q(w_n^*, w^*) \end{array} \right\} \quad (2.17)$$

$$q\left(S_n(w_n^*), S(w_n^*)\right) \le \left[\frac{ra(1+a)}{\left(1-c-r^2 d\right)}\right] q(w_n^*, w^*) + \left[\frac{1-c}{1-c-r^2 d}\right]\left[\left(\frac{rd}{1-c}\right) + r^2\right] q\left(S(w^*), S_n(w^*)\right) \quad (2.18)$$

Using (2.18) in (2.11) yields:

$$q(w_n^*, w_n^*) \le [\frac{(1-c+d)r}{1-c-ra}] \left\{ [\frac{ra(1+a)}{\left(1-c-r^2 d\right)}] q(w_n^*, w^*) + [\frac{1-c}{1-c-r^2 d}][(\frac{rd}{1-c}) + r^2]\, q\left(S(w^*), S_n(w^*)\right) \right\}$$

$$\le [\frac{(1-c+d)r}{1-c-ra}][\frac{ra(1+a)}{(1-c-r^2 d)}] q(w_n^*, w_n^*) + [\frac{(1-c+d)r}{1-c-ra}][\frac{1-c}{1-c-r^2 d}][(\frac{rd}{1-c}) + r^2] q\left(S(w^*), S_n(w^*)\right)$$

Consequently:

$$q(w_n^*, w^*) \le [\frac{1}{1-\left[\frac{(1-c+d)r}{1-c-ra}\right]\left[\frac{ra(1+a)}{1-c-r^2 d}\right]}] \left[\frac{(1-c+d)r}{1-c-ra}\right] \left[\frac{1-c}{1-c-r^2 d}\right]\left[\left(\frac{rd}{1-c}\right) + r^2\right] q(S(w^*), S_n(w^*))$$

Taking the limit as $n \to \infty$ yields that $\lim_{n\to\infty} q(w_n^*, w^*) = 0$ because of the given assumption $\lim_{n\to\infty} q\left(S_n(w^*), S(w^*)\right) = 0$. This completes the proof.

Conclusion

This paper adjusts conditions on new defined contraction type of mapping; namely {*a*, *d*, *c*, *r*}-ctype on quasi metric space, checked the validation of existence of unique fixed point of such type, gives a generalization of theorem (1) with these adjustments, shows that Corollary (1) extends some of the results given in (El-Shobaky *et al*., 2007; Gregus, 1980; Wong, 1975) and then builds a convergence theorem for sequence of fixed points of $\{S_n\}_{n\in N}; \{w_n^*\}_{n\in N}$ to the unique fixed point of *S*; *w* under the as-sumption $\lim_{n\to\infty} S_n(w) = w$.

Acknowledgment

Sincer thanks to the reviwer valable comments.

Conflict of Interests

The author has no interesting conflict of interest.

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