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Bayesian Estimation for Lomax Distribution: A Comparison of Loss Functions Using Jeffreys Priors

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Abstract: This study investigates the estimation of the shape parameter for the Lomax distribution using complete data. We compare the performance of the classical Maximum Likelihood Estimation (MLE) method against the Bayesian framework. Within the Bayesian approach, three distinct loss functions were utilized: the linear exponential (LINEX), general entropy, and weighted general entropy loss functions. The precision of these estimators was assessed through Mean Squared Error (MSE) and bias metrics. Monte Carlo simulation results demonstrate that the LINEX loss function consistently provides the most accurate parameter estimates, yielding the lowest MSE and bias values.

Keywords: Lomax Distribution, Maximum Likelihood Estimation, Bayesian Approach, Linear Exponential, Weighted General Entropy, Mean Squared Errors

Introduction

The Lomax Distribution (LD), also known as the Pareto type-II distribution, is a continuous distribution introduced by Lomax (1954). It finds applications in various fields, such as lifetime and reliability modelling, medicine, biology, and economics. The Lomax distribution is a special case of the Pearson type VI distribution and is a mixture of Exponential and Gamma distributions. For further details on the properties of LD and their applications, please refer to (Alnssyan, 2023; Marshall and Olkin, 2007; Bryson, 1974). The Lomax distribution is given by two parameters: σ (scale parameter) and β (shape parameter). Its Probability Density Function (PDF) is given by the following:

$$f(X;\beta,\sigma) = \frac{\beta}{\sigma}\left(1+\frac{X}{\sigma}\right)^{-(\beta+1)}, X \geq 0, (\beta,\sigma) > 0 \qquad (1)$$

Several researchers have estimated two parameters for this distribution. For instance, Pak and Reza (2018) applied the maximum likelihood estimate of the parameters using Newton–Raphson, the EM algorithm, and Tierney and Kadane's approximation. Labban (2019) presented the moment, maximum likelihood, and term omission methods to compare the best estimation methods for LD parameters. Kumari and Kumar (2022) discussed the entropy and precautionary loss functions and the extension of the Jeffreys and gamma priors to compute the best estimation method for the two LD parameters. The E-Bayesian method was used to compute the estimates of unknown parameters, as well as certain survival time parameters, such as reliability and hazard functions for the Lomax distribution, based on type-II censored data by Okasha (2014).

Both expected Bayesian and Bayesian estimation methods were applied to estimate the LD's shape parameters. For example, Al-Bossly (2021) employed the weighted composite linear exponential loss function (LINEX) loss function, while Liu and Zhang (2022) applied the square error loss, K-loss function, and entropy loss function. Fitrilia *et al*. (2018) studied the likelihood function and expected Bayesian methods to estimate the shape parameter of an LD, using right-censored type II data. In addition, a balanced squared error loss function was utilized based on type-II censored data by Okasha (2014). Nasiri and Hosseini (2012) computed Bayesian and classical estimations to study the parameters based on the recorded values for LD. IJAZ (2021) discussed the squared error loss function, quadratic loss function, weighted squared error loss function, and precautionary loss function estimation methods under uniform and Jeffrey priors by applying engaging data to estimate LD's parameter. Cramer and Schmiedt (2011) discussed maximum likelihood estimation, Fisher information matrix, and optimal Fisher information to estimate the unknown parameters of type-II censored data from LD.

The Bayesian method has been applied to estimate the unknown parameters for different distributions, such as the Weibull, exponential, and gamma distributions (Naji and Rasheed, 2019a-b; Ni and Sun, 2021; Annan, 2015; Yadav *et al.*, 2019; Smith and Naylor, 1987). A new loss function called the Weighted General Entropy loss function (WGE) was first proposed by Al-Duais. (2022) to be the best estimation of the Weibull distribution method's reliability, compared to the Maximum Likelihood Estimation (MLE), squared error loss function, and general entropy loss function.

Mohammed *et al.* (2022) demonstrated that the LINEX loss function is the best estimation method for the mean, gamma distribution, and Poisson process compared to MLE. The LINEX loss function is the best estimation method for the censored data based on the Exponential distribution with cure fraction as reported by Solimana *et al.* (2006) discussed that for the case of the unknown shape and scale parameters of the Weibull distribution, the LINEX loss function was the better estimation method.

The Bayes estimator under the squared error loss function of the shape parameter of the Lomax distribution has been studied by IJAZ (2021). The study of the shape parameter of the Lomax distribution plays an important role in the analysis and study of the failure rate in life tests in medicine, biology, and engineering. In this study, we considered the maximum likelihood estimation, symmetric and asymmetric loss functions, such as the linear exponential loss function, general entropy loss, and weighted general entropy loss functions, to determine the best estimation of the shape parameter of the LD. We suppose that σ is a known parameter and equals 1, so the PDF of LD can be written as follows:

$$f(X;\beta) = \beta(1+X)^{-(\beta+1)},\ X \ge 0, \beta > 0 \tag{2}$$

Materials and Methods

This section presents methods for estimating the shape parameters of the Lomax distribution based on MLE, LINEX, General Entropy (GE), and WGE.

Maximum Likelihood Estimation (MLE)

Let $X = X_1, X_2, X_n$ be a set of random variables based on the Lomax distribution. The likelihood of β can be described as:

$$L(X|\beta) = \prod_{i=1}^{n} \beta(1+X_i)^{-(\beta+1)} = \beta^n exp[-(\beta+1)\eta] \tag{3}$$

Where $\eta = \Sigma_{i=1}^{n} ln(1+X_i)$.
The logarithm of the likelihood function is:

$$lnL(X|\beta) = n\ln(\beta) - (\beta+1)\sum_{i=1}^{n} \ln(1+X_i)$$

Therefore, the MLE estimator of β is obtained by solving the following equation with respect to β:

$$\frac{\partial InL(X|\beta)}{\partial\beta} = 0 \Rightarrow \frac{n}{\beta} - \sum_{i=1}^{n} In(1+X_i) = 0$$

Then, the Maximum Likelihood Estimation (MLE) of β is given by:

$$\hat{\beta}_{MLE} = \frac{n}{\Sigma_{i=1}^{n} In(1+X_i)} \tag{4}$$

Bayesian Estimation (BE)

Here, we derive the Bayes estimation of the shape parameter, β, of the Lomax distribution. For this purpose, we used the LINEX, general loss function (GE), and weighted general loss functions (WGE). The Jeffreys prior is used as a prior distribution, which is the square root of the determinant of the Fisher information matrix and is given by:

$$g(\beta) = k\frac{1}{\beta} \tag{5}$$

Where k is a constant. By combining the likelihood function in Eq. 3 with the prior of β in Eq. 5, we get the posterior of β as:

$$\begin{aligned}\pi(X,\beta) &= \frac{k\frac{1}{\beta}\prod_{i=1}^{n}(1+X_i)^{-(\beta+1)}}{\int_0^\infty k\frac{1}{\beta}\prod_{i=1}^{n}(1+X_i)^{-(\beta+1)}d\beta}\\ &= \frac{\frac{1}{\beta}\beta^n \exp[-(\beta+1)\eta]}{\int_0^\infty \frac{1}{\beta}\beta\exp[-(\beta+1)\eta]\,d\beta}\\ &= \frac{\beta^{n-1}\eta^n\exp(-\eta\beta)}{\Gamma(n)}\end{aligned} \tag{6}$$

Examples of linear and exponential loss functions, including linear exponential, general entropy, and weighted general entropy, are discussed below to estimate the Lomax distribution's parameters.

Estimates Based on Linear Exponential Loss Function (LINEX)

The LINEX loss function is an asymmetric loss function, which was discussed by Zellner (1983) and is expressed as:

$$L(\Delta) = \exp(a\Delta) - a\Delta - 1, a \neq 0$$

Where: $\Delta = \hat{\beta} - \beta$. The shape of this loss function is determined by the value of a. In this case when $a > 1$ that means overestimation and underestimation when $a < -1$ If a is close to zero, the linear exponential loss function approaches the squared-error loss function, which is a symmetric loss function. For more details on the LINEX loss function, see Ahmad *et al.* (2021). The $\hat{\beta}_{BL}$ is the estimator of β under LINEX and can be shown as follows:

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$$\begin{aligned}\hat{\beta}_{BL} &= -\frac{1}{a} In\,(E\left(\exp(-a\beta)\right))\\ &= -\frac{1}{a} In\,[\int_0^{\infty} \exp(-a\beta)\,\pi(X,\beta)\,d\beta]\\ &= -\frac{1}{a} In\,[\int_0^{\infty} \exp(-a\beta)\,\frac{\beta^{n-1}\eta^n \exp(-\eta\beta)}{\Gamma(n)} d\beta]\\ &= \frac{n}{a} In\,(1+\frac{a}{\eta})\end{aligned} \quad (7)$$

Estimates Based on General Entropy (GE)

GE is an asymmetric loss function that can be written as (Ghosh and Yang, 1988; Dey, 1992; Al-Duais, 2022):

$$L(\hat{\beta},\beta) \propto (\frac{\hat{\beta}}{\beta})\ \ q - q\ln(\frac{\hat{\beta}}{\beta}) - 1,\ q \neq 0$$

The Bayes estimator of β, under the GE loss function, is denoted by $\hat{\beta}_{GE}$ and is given by:

$$\begin{aligned}\hat{\beta}_{GE} &= [E(\beta^{-q})]^{-\frac{1}{q}}\\ &= [\int_0^{\infty} \beta^{-q}\,\pi(X,\beta)\,d\beta]^{-\frac{1}{q}}\\ &= [\int_0^{\infty} \beta^{-q}\,\frac{\beta^{n-1}\eta \exp(-\eta\beta)}{\Gamma(n)}]^{-\frac{1}{q}}\\ &= [\frac{\eta^q \Gamma(n-q)}{\Gamma(n)}]^{-\frac{1}{q}}\end{aligned} \quad (8)$$

Estimates Based on Weighted General Entropy (WGE)

The weighted general entropy loss function was used to estimate parameters in Al-Duais (2022). It was created by determining the weight of the general entropy loss function. The loss function depends on the WGE loss function, which can be expressed as:

$$L(\hat{\beta},\beta) \propto \omega(\beta)\,[\frac{\hat{\beta}}{\beta} - q\ln(\frac{\hat{\beta}}{\beta}) - 1],\ q \neq 0$$

Where: $\omega(\beta) = \frac{1}{\beta^{\zeta}}$, ζ is a constant and $\zeta \neq 0$.

The Bayes of the shape parameter of the Lomax distribution, β, using the WEG loss function is denoted as: $\hat{\beta}_{WGE}$, and it is given as:

$$\hat{\beta}_{WGE} = [\frac{E(\beta^{-(\zeta+q)})}{E(\beta^{-\zeta})}]^{-\frac{1}{q}}$$

Let suppose $I_1 = E\left(\beta^{-(z+q)}\right)$ and $I_2 = E(\beta^{-z})$ Then we can get:

$$\begin{aligned}I_1 &= \int_0^{\infty} \beta^{-(\zeta+q)}\,\frac{\beta^{n-1}\eta^n \exp(-\eta\beta)}{\Gamma(n)} d\beta\\ &= \frac{\eta^{\zeta+q}}{\Gamma(n)}\Gamma(n-\zeta-q)\end{aligned} \quad (9)$$

From Eq. 8, it is noted that:

$$\begin{aligned}I_2 &= E(\beta^{-\zeta})\\ &= [\frac{\eta^{\zeta}\Gamma(n-\zeta)}{\Gamma(n)}]\end{aligned} \quad (10)$$

Then from Eqs. 9-10, one can get:

$$\hat{\beta}_{WGE} = [\frac{\eta^q \Gamma(n-q-\zeta)}{\Gamma(n-\zeta)}]^{-\frac{1}{q}} \quad (11)$$

Note that GE is a special case of WGE when $\zeta = 0$.

Simulation Study

In this section, we examine the performance of the ML-Bayesian method using Jeffreys before estimating the Lomax distribution parameters. The Mean Squared Error (MSE) was calculated to compare the estimation methods. A Monte Carlo simulation was performed according to the following steps. Generate random variables X_i, where: $X_i = \sigma[(1-U_i)^{(-\frac{1}{\beta})} - 1]$ where: $X_i = X_1, X_2 \ldots, X_n$, and we have selected the sample size n = 25, 75, and 100. The random variable U_i is distributed as a Uniform distribution [0, 1]. To observe the effect of the shape parameter of the Lomaix distribution on the estimation, two different values of β were chosen as β = 2 and 3, and σ is known and equals 1.

The maximum likelihood estimation, $\hat{\beta}_{PMLE}$, from Eq.4, is used to estimate the shape parameter of the Lomax distribution. The Bayesian with Jeffreys prior under linear exponential loss function (LINEX) computed in Eq.7 was used to estimate the shape parameter of the Lomax distribution. The values of the LINEX constant were selected as a = 0.5 and 1.5. The Bayesian method with the Jeffreys prior under the general entropy loss function given in Eq.8 was applied to estimate the shape parameter of the Lomax distribution. The value of the GE constant was selected to be $q = \pm 2$. The Bayesian with Jeffreys prior under the Weighted General Entropy loss function (WGE) from Eq.11 was used to estimate the shape parameter of the Lomax distribution. The values of the WGE constants were selected to be $q = \pm 2$ and $z = \pm 1$. The number of replicates was set to R = 100,000. For each repetition, the Mean Squared Error (MSE) and bias were used to compare the simulation estimates of the different estimation methods:

$$MSE = \sum_{i=1}^{R} \frac{(\hat{\beta} - \beta)^2}{R}$$

And:

$$Bias = \sum_{i=1}^{R} \frac{|\hat{\beta} - \beta|}{R}$$

The computational results are listed in Tables 1 and 2.

Results and Discussion

As Tables 1 and 2 show, the estimated values, mean square error, and bias using MLE, BL, GE, and WGE estimation methods were computed for the true parameter β = 2 or 3, respectively. Figures (1-4) show the behavior of mean square error and bias of the estimator methods for different values of β with different sample sizes, graphically. The following observations can be concluded. As the sample size increases, the estimated values of β are very close to the real values.

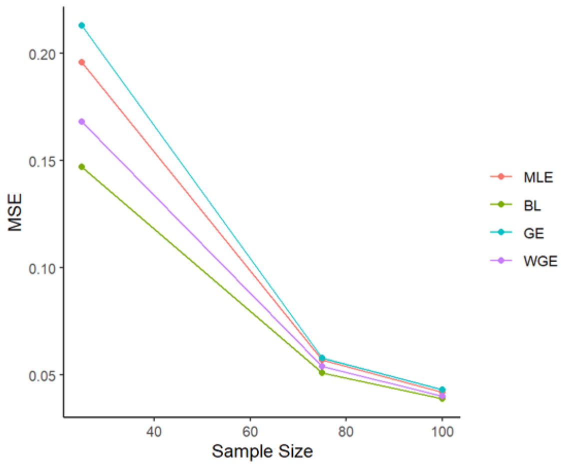

Fig. 1: The graph of MSEs of different estimators of β = 2

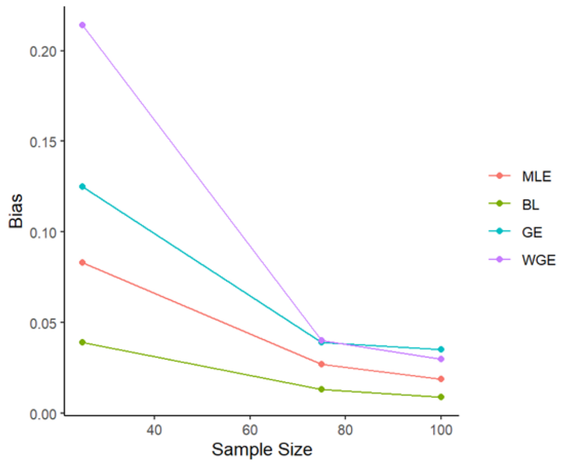

Fig. 2: The Graph of Bias of different estimators of β = 2

Table 1: Estimated values, MSE, and bias of β = 2 for LD, where n = 25, 75, and 100

n	Estimators	Estimated values	MSE	Bias
	MLE	3.0371	0.1965	0.0835
	BL(a=0.5)	1.7956	0.1738	0.0402
	BL(a=1.5)	2.4757	0.1479	0.0395
25	GE(q=-2)	3.0785	0.2131	0.1253
	GE(q=2)	1.6780	0.1685	0.0401
	WGE(q=-2, Z=-1)	2.0185	0.2537	0.2067
	WGE(q=2, Z=1)	2.3600	0.1682	0.2148
	MLE	1.9435	0.0573	0.0276
	BL(a=0.5)	2.0897	0.0551	0.0139
	BL(a=1.5)	2.0897	0.0519	0.0135
75	GE(q=-2)	2.4628	0.0585	0.0398
	GE(q=2)	2.2158	0.0539	0.0132
	WGE(q=-2, Z=-1)	1.9526	0.0634	0.0681
	WGE(q=2, Z=1)	1.6259	0.0541	0.0404
	MLE	2.0499	0.0422	0.0196
	BL(a=0.5)	2.0245	0.0410	0.0098
	BL(a=1.5)	2.3052	0.0395	0.0097
100	GE(q=-2)	2.3799	0.0433	0.0356
	GE(q=2)	1.9697	0.0403	0.0099
	WGE(q=-2, Z=-1)	2.0368	0.0453	0.0516
	WGE(q=2, Z=1)	2.3093	0.0404	0.0308

Table 2: Estimated values, MSE, and bias of β = 3 for LD, where n = 25, 75, and 100

n	Estimators	Estimated values	MSE	Bias
	MLE	2.5119	0.4417	0.1282
	BL(a=0.5)	2.6225	0.3733	0.0302
25	BL(a=1.5)	2.8771	0.3127	0.1432
	GE(q=-2)	2.6565	0.4790	0.1912
	GE(q=2)	2.0265	0.3812	0.0646
	WGE(q=-2, Z=-1)	2.9088	0.5717	0.3093
	WGE(q=2, Z=1)	2.1838	0.3777	0.1867
	MLE	2.6658	0.1285	0.0404
	BL(a=0.5)	2.7643	0.1217	0.0091
75	BL(a=1.5)	2.3231	0.1141	0.0476
	GE(q=-2)	2.7061	0.1323	0.0624
	GE(q=2)	2.2085	0.1219	0.0212
	WGE(q=-2, Z=-1)	2.7859	0.1421	0.1017
	WGE(q=2, Z=1)	2.8859	0.1225	0.0612
	MLE	2.7878	0.0948	0.0309
	BL(a=0.5)	2.9218	0.0908	0.0077
100	BL(a=1.5)	2.8895	0.0871	0.0308
	GE(q=-2)	2.9999	0.0961	0.0442
	GE(q=2)	2.7264	0.0912	0.0142
	WGE(q=-2, Z=-1)	2.9500	0.1028	0.0748
	WGE(q=2, Z=1)	2.8636	0.0911	0.0450

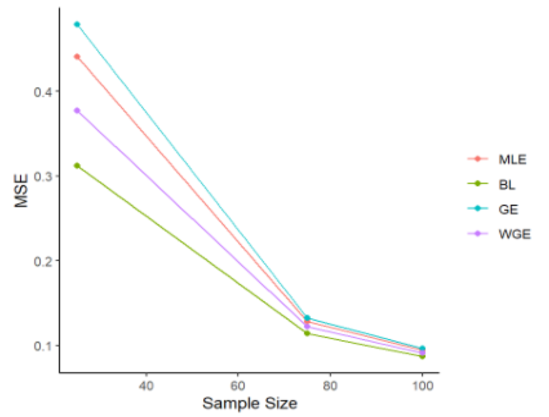

Fig. 3: The Graph of MSEs of different estimators of β = 3

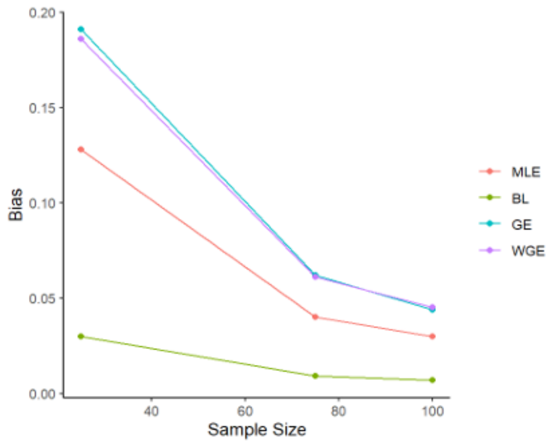

Fig. 4: The Graph of Bias of different estimators of β = 3

The results show that MSE and bias values decrease as the sample size increases for all estimation methods. For all sample sizes, the values of MSE and bias increase as the parameter value increases for all estimation methods of β. The Bayesian estimators under different loss functions performed better than the maximum likelihood estimation method, with the small values of MSE for different sample sizes and different values of the true parameter, which are similar results in Naji and Rasheed (2019a-b); they found that the generalized weighted loss function and precautionary loss function were better than the MLE. The LINEX loss function is a better estimation method compared to all other estimates because it has the smallest values of the MSE for all sample sizes. The results are similar to the. Findings of Mohammed *et al*. (2022), who concluded that the LINEX loss function was a better estimator than other estimation methods. Compared with the results by IJAZ (2021); Kumari and Kumar (2022); Al-Bossly (2021), they found that the Precautionary Error Loss Function (PELF) and Weighted Composite Linex Loss Function (WCLLF) have better results in estimating the Lomax parameters compared with other estimation methods.

Application on Real Data

To illustrate the estimation methods discussed here, we studied data taken from Stablein *et al.* (1981) and reported in Bekker *et al.* (2000). This dataset considers the survival times in years for 45 patients affected by gastric cancer and is given as follows: 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.570, 0.641, 0.644, 0.696, 0.841, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.4853, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458.

To test the following hypothesis:

H_0: The data set ~ follows the Lomax distribution

Vs

H_A: The data set ↓~ Lomax distribution

Table 3 shows Anderson-Darling (AD), Cramer-Von Mises (CvM), and Pearson χ 2 tests with the corresponding p-values of the dataset. The p-values of the tests were greater than the significance level (0.05), we failed to reject the null hypothesis, and conclude that the dataset followed the Lomax distribution. Table 4 confirms the analysis results that the Lomax distribution is more suitable for the data than other distributions, with the smallest values of AD and CvM. Figure 5 represents the empirical and fitted survival functions, which show that the fitted Lomax survival function is near the 45-degree line. That supports the suitability of these data for the Lomax distribution.

Table 3: Table of test statistic

Test	statistic	p-value
AD	1.7945	0.1276
CvM	0.2777	0.1564
Person χ^2	13.1304	0.1568

Table 4: Test statistics of Cauchy, Logistic, and Normal distributions

Test	Cauchy	Logistic	Normal
CvM	0.6985	0.3102	0.4727
AD	4.5255	2.3258	2.7888

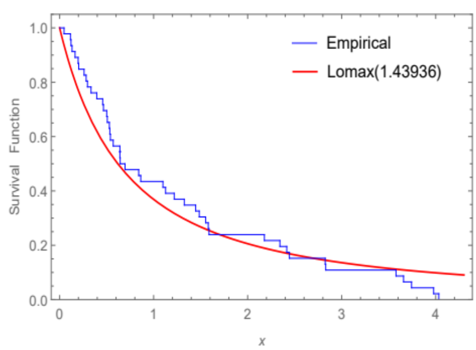

Fig. 5: The empirical and survival function plot of the gastric cancer patients' data set

To illustrate its usefulness and compare its goodness-of-fit with other invariant forms of the Lomax distribution, including Cauchy, Logistic, and Normal distributions, using the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). The formulas are as follows.

AIC is defined by:

$$AIC = 2k - 2\log L(\hat{\beta}) \quad (12)$$

BIC is expressed as:

$$BIC = -2\log L(\hat{\beta}) + k\log L(n) \quad (13)$$

Where *L* refers to the likelihood under the fitted model, *k* is the number of parameters in the model, and *n* is the sample size.

The small values of the model's AIC and BIC indicate that the mean square error value is also small. This indicates that the model is more accurate than the other models. The model with the minimum values for these criteria was selected as the best model. Further information on AIC and BIC can be found in (Fabozzi *et al.*, 2014; Acquah *et al.,* 2013).

Table 5 presents Log-Likelihood (LL), Akaike Information Criteria (AIC), and Bayesian Information Criteria (BIC) values of Cauchy, Logistic, Normal, and Lomax distributions. According to the results, it is noted that the Lomax distribution has the lowest AIC and BIC of the real data set, indicating a better fit compared to other models. When focusing on the values of log-likelihood, the higher the value of the log-likelihood, the better a model of the real data set. As noted, the Lomax distribution has the highest LL compared with the values of the log-likelihood of Cauchy, Logistic, and Normal distributions. We can conclude that the Lomax distribution appears to be a better fit for this data set than other models.

Table 5: Goodness of fit criteria AIC and BIC of Cauchy, logistic, and normal distributions

The distribution	LL	AIC	BIC
Cauchy	-73.1888	150.3776	154.0349
Logistic	-71.9136	147.8272	151.4844
Normal	-72.1444	148.2892	151.9464
Lomax	-61.2054	128.4108	133.8967

Conclusion

In this study, we compared the estimated values of the Lomax distribution's shape parameters using both the maximum likelihood estimator and Bayesian estimation. Using Jeffrey's prior, we evaluated these methods under three different loss functions: LINEX loss, general entropy loss, and weighted general loss. We focused on minimizing the Mean Squared Error (MSE). Based on the simulation study, we noted that the Bayesian estimation of the shape parameter of the distribution under the LINEX loss function with a value of 1.5 has the best performance with the smallest MSE values. In conclusion, application to real data shows that the Lomax distribution is better than the Cauchy, Logistic, and Normal distributions. Subsequent research efforts may focus on exploring the estimation challenges of the proposed distribution if both parameters are unknown. Additionally, a comparison could be made between the precautionary loss function, the weighted composite LINEX loss function, and the LINEX loss function.

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Ethics

The author mentioned in the paper read, approved, and gave consistent comments. We approved that it is not under review for publication, and it is not accepted for publication in another journal.

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